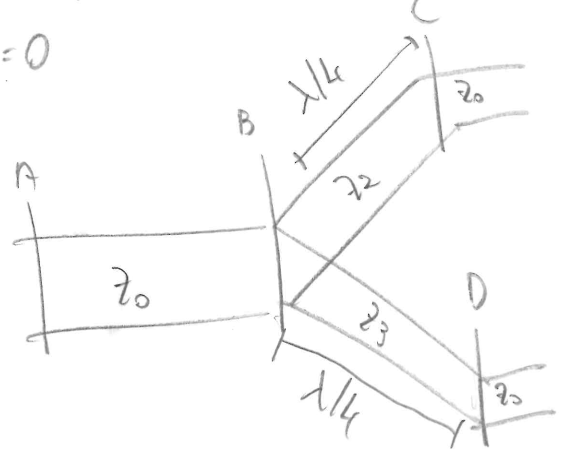
Tema di esame

1) Divisore di potenza $Z_0 = 50\,\Omega$ $\varepsilon_r = 4$ $h = 1mm$ adattato a ingresso tale che:

- $V_2 = -0{,}5\, V_1$ → data questa, uso il T-junction "variante":
- $\Gamma_1 = 0$



Calcolo $\dfrac{V_{C^+}^+}{V_{A^-}^+}$:

$$V_{C^+}^+ = V_{C^-}^+ (1+\Gamma_{C^-}) = V_{B^+C}^+ \exp(-jkl)(1+\Gamma_{C^-}) =$$

$$= V_{B^-}^+ \frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}} (1+\Gamma_{C^-}) \exp(-jkl);$$ mi fermo qui.

↳ $V_{C^+}^+ = -j\, \xi_{C^-} V_{B^-}^+$;

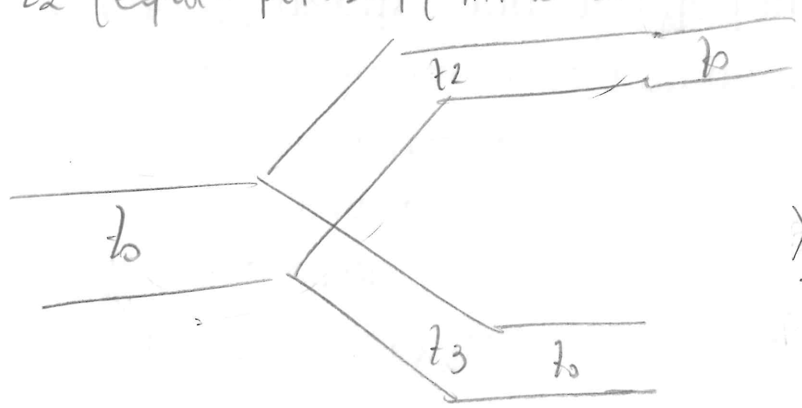
Allo stesso modo, $V_{D^+}^+ = -j\, \xi_{D^-} V_{B^-}^+$ (passaggio fondamentale)

Ma: $\xi_{C^-} = \dfrac{Z_0}{Z_2}$; $\xi_{D^-} = \dfrac{Z_0}{Z_3}$;

↳ $\boxed{\dfrac{V_{C^+}^+}{V_{D^+}^+} = \dfrac{Z_3}{Z_2}}$;

una nota: se io volessi che $\dfrac{V_C}{V_D} = -1$, devo aggiungere un $\exp(-jkl_3)$ tale da non cambiar nulla, se non il segno.

$Z_3 = Z_2$ (equi-partisco), ma lo schema sarà:



$Z_{00_2} = \sqrt{Z_0 Z_2} = 70{,}71\,\Omega$

$Z_{00_3} = \sqrt{Z_0 Z_3}$

$\dfrac{\lambda_g}{4} = \dfrac{c}{f\sqrt{\varepsilon_{eff}}\, 4}$

↓

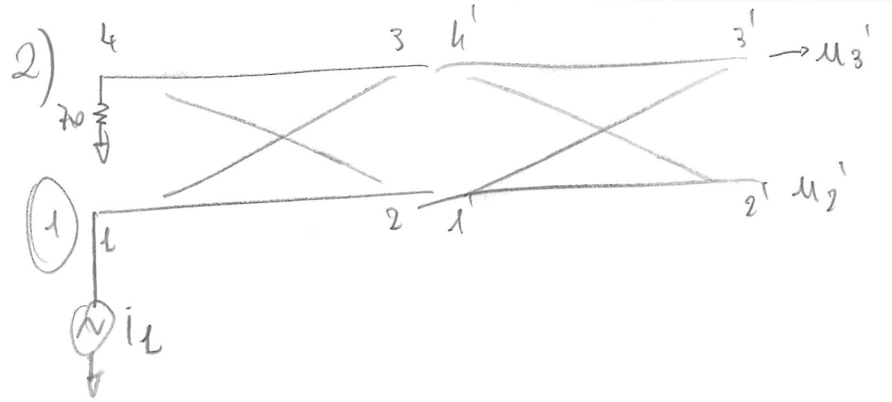
$Z_3 = Z_2 = 100\,\Omega$;

$l_2 = \lambda/4 + \lambda/2$;

$l_3 = \lambda/4$;

Z_{00}	w/h	ε_{eff}	$l = \lambda/4$	$l = 3\lambda/4$	w
50	2,053	3,073			
100	0,695	2,799	14,94	44,83	
70,71	1,102	2,935	14,59		

2) (2)



$$\underline{\underline{S}} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix} \frac{1}{\sqrt{2}}$$

$u_3' = i_1 \left[j\beta\alpha + \alpha j\beta \right]$; ora, ricordo che:

$u_4' = i_1 \left[\alpha\alpha + j\beta j\beta \right]$; $C = -20 \log_{10}(\beta)$

$\hookrightarrow \beta = 10^{-\frac{C}{20}} = 0{,}3828$;

$u_3' = 2j i_1 \cdot 0{,}3537$;

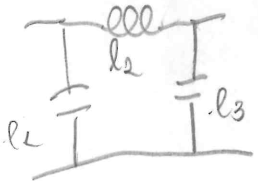
$\alpha = \sqrt{1-\beta^2} = 0{,}9238$;

$u_4' = i_1 \left[0{,}853 - 0{,}1464 \right] = 0{,}7065\, i_1$;

3) Filtro taglio 2GHz, inizio banda @ 4GHz; P_{LR} @ 2GHz $\leq$ 15 dB;
(diciamo Butterworth perché è il meglio). $Z_0 = 50\,\Omega$.

$\frac{4}{2} = 2 \to \Omega - 1 = 1 \to \boxed{N = 3}$; $g_1 = g_3 = 1$; $g_2 = 2$; $g_4 = 1$.

Filtro prototipo:



Scelgo a π per minimizzare il numero di stub serie, che verran invertiti.

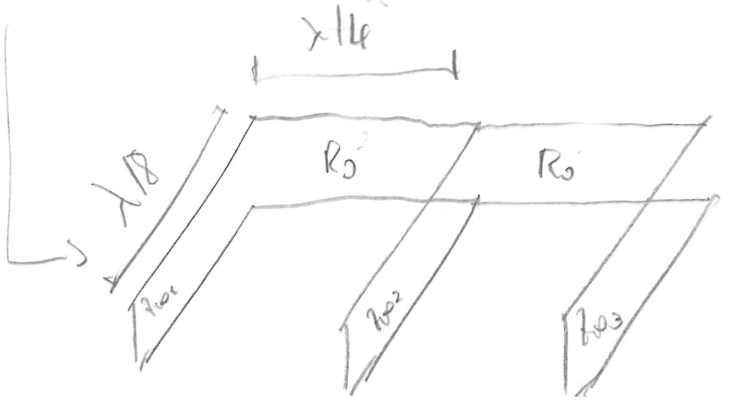
Ricordo che: $e_i = \frac{g_m}{R_0}$; $l_i = g_m R_0$;

Allora: vediamo come si trasforma il C!

$j\omega e_n \to -j \frac{\omega_0}{\omega} e_n = -j \frac{\omega_0}{\omega} \frac{g_m}{R_0} = \frac{1}{j\omega L}$, $\frac{1}{L} = \frac{\omega_0 g_m}{R_0} \to L = \frac{R_0}{\omega_0 g_m}$

Applicando la trasformazione di Richards:

$Z_{0} = \omega_0 L_n = \frac{R_0}{g_m}$



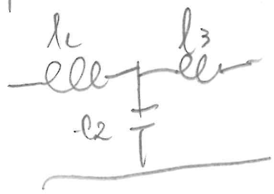
$Z_{01} = 50\,\Omega$

$Z_{02} = 25\,\Omega$

$Z_{03} = 50\,\Omega$

"4") Filtro Butterworth, $N = 3$; $Z_0 = 50\Omega$; $g_1 = g_3 = 1$, $g_2 = 2$; $f_c = 8GHz$

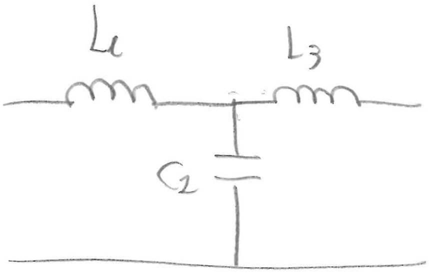
passa basso:



$$\begin{cases} l_i = g_m R_0 \\ c_i = \frac{g_m}{R_0}; \end{cases} \longrightarrow$$

$j\omega l_i \rightarrow j \frac{\omega}{\omega_0} g_m R_0 = j\omega L_i$; $L_i = \frac{g_m R_0}{\omega_0}$;

$j\omega c_i \rightarrow j \frac{\omega}{\omega_0} \frac{g_m}{R_0}$; $C_i = \frac{g_m}{\omega_0 R_0}$;



$L_1 = 0{,}99\ nH$

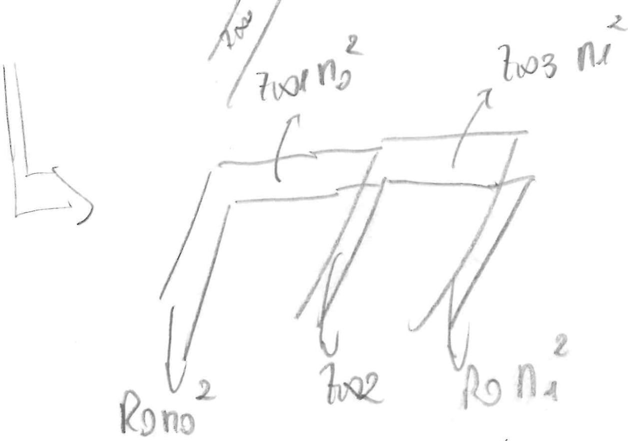
$C_2 = 0{,}796\ pF$

$L_3 = L_1$

Applico la trasf. di Richards:

$Z_{0L} = L\omega_c = g_m R_0$; $n_0^2 = \frac{R_0}{Z_{01}} + 1$

$Z_{0C} = \frac{1}{C\omega_c} = \frac{R_0}{g_m}$; $n_1^2 = \frac{R_0}{Z_{03}} + 1$



$Z_{0C1} = Z_{03} = 50\Omega$;

$Z_{0C2} = 25\Omega$;

$n_0^2 = 1 + 1 = 2$

$n_1^2 = 1 + 1 = 2$

Z_0	$\frac{W}{h}$	ϵ_{eff}	$\frac{c}{f\sqrt{\epsilon_{eff}} \cdot 4}$	W
~~50Ω~~				
100Ω	0,495	2,709	3,6 mm	2,475 mm
25Ω	2,053	3,073	3,348 mm	10,27 mm

Esercizi in aula microonde (1)

Calcolare lungh. e largh. di una microstriscia con $\vartheta = \beta l = 90°$, $d = 1{,}27$ mm, $\varepsilon_r = 2{,}2$, $Z_0 = 50\,\Omega$, $f = 2{,}5$ GHz, $\frac{W}{d} = 3{,}173$; "$h \equiv d$"

da grafico $\frac{W}{d} \approx 3{,}2$; dal grafico, $\sqrt{\varepsilon_{eff}} \approx 1{,}35 \rightarrow \varepsilon_{eff} \approx 1{,}8$.

$W = \frac{W}{h} \cdot d = 4{,}064$;

$\beta = k_0 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}\sqrt{\varepsilon_{eff}}\, l = \frac{\pi}{2} \rightarrow l = \frac{c}{4f\sqrt{\varepsilon_{eff}}} = 22{,}22$ mm

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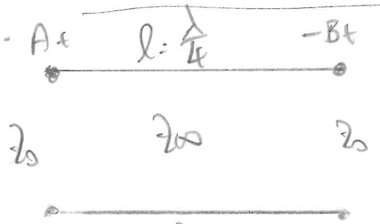


$Z_0 = 100\,\Omega$ Calcolare $\underline{\underline{S}}$

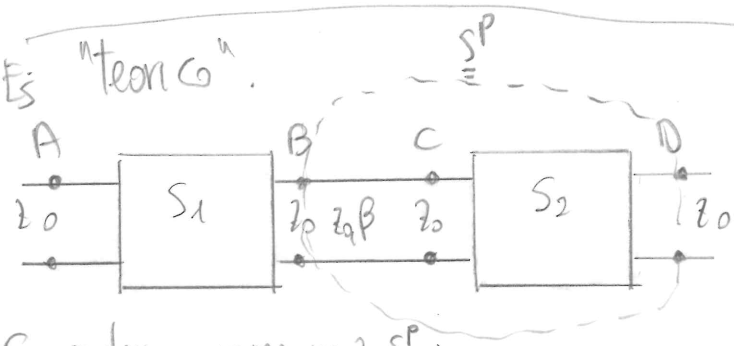
$Z_{00} = 50\,\Omega$

$\Gamma_{B^-} = \frac{Z_0 - Z_{00}}{Z_0 + Z_{00}} =$ $V^+_{B^+} = V^+_{B^-}\frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}} = V^+_{A^+}\exp(-jkl)\frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}} = V^+_{A^-}\exp(-jkl)\frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}}\frac{1+\Gamma_{A^-}}{1+\Gamma_{A^+}}$

$\Rightarrow -0{,}8j = S_{21}$; per reciprocità & simmetria: $\underline{\underline{S}} = \begin{bmatrix} -0{,}6 & -j0{,}8 \\ -j0{,}8 & -0{,}6 \end{bmatrix}$

[si poteva anche notare che $\frac{1+\Gamma_{B^-}}{1+\Gamma_{A^+}} = \frac{1-\Gamma_{A^+}}{1+\Gamma_{A^+}} = y_{A^+}$]

Es "teorico".



$\underline{\underline{S}}_1 = \begin{bmatrix} S^1_{11} & S^1_{12} \\ S^1_{21} & S^1_{22} \end{bmatrix}$ $\underline{\underline{S}}_2 = \begin{bmatrix} S^2_{11} & S^2_{12} \\ S^2_{21} & S^2_{22} \end{bmatrix}$

$\underline{\underline{S}}_{eq} = ?$

Considero prima una $\underline{\underline{S}}^P$;

$S^P_{11} = \frac{b_1}{a_1}\Big|_{a_2=0} = \Gamma_B$; si ricordi che:

$\Gamma_C = \frac{S^2_{21}S^2_{12}\Gamma_D}{1-\Gamma_D S^2_{22}} + S^2_{11}$; $\Gamma_D = 0$ dal momento che carico su Z_0

$\hookrightarrow \Gamma_C = S^2_{11}$; $\Gamma_B = S^2_{11}\exp(-j2kl_{BC})$ [dal momento che è <u>tutto adattato</u>] [Γ si trasporta secondo $\exp(-j2kl)$]

$S^P_{21} = \frac{V^+_{D^+}}{V^+_{B^-}}$; $V^+_{D^+} = V^+_{D^-} = \sqrt{Z_0}\, b_2$;

$b_2 = S^2_{11}a_1 + S^2_{12}a_2$ [ma $a_2 = 0$ per adattamento]

$\hookrightarrow b_2 = S^2_{11}a_2$; $\rightarrow a_1 = \frac{V^+_{C^+}}{\sqrt{Z_0}}S^2_{11} = \frac{V^+_{D^-}}{\sqrt{Z_0}} \rightarrow V^+_{D^+} = V^+_{C^+}S^2_{11}$;

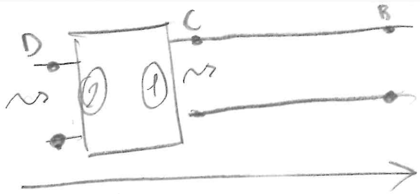
Si ha adattamento, dunque:

$V^+_{D^+} = V^+_{B^+}S^2_{11}\exp(-jkl_{AB}) \rightarrow S^P_{21} = S^2_{11}\exp(-jkl_{AB})$

$S_{22} = \Gamma_{D^-} = \Gamma_D = \frac{b_2}{a_2}\Big|_{a_1=0}$; uso il trasporto dei Γ:

$\Gamma_{B^+} = 0$; $\Gamma_{C^-} = 0$; $\Gamma_{D^+} = \Gamma_{D^-} = S^2_{22} + \frac{S^2_{21}S^2_{12}\Gamma_C}{1-\Gamma_C S^2_{11}} = S^2_{22}$

Calcolo S_{12}^P:



(2)

$$S_{12}^P = \frac{b_1}{a_2}\Big|_{a_1=0} = \frac{V_{B^+}^+}{V_{D^-}^+}$$

$V_{B^+}^+ = V_{B^-}^+ = V_{C^+}^+ \exp(-jkl_{CB})$; $b_1 = \cancel{a_1 S_{11}^2} + a_2 S_{12}^2$

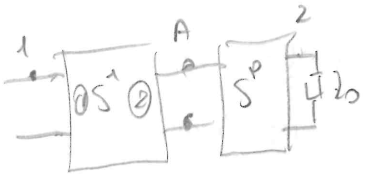
$\hookrightarrow = a_2 S_{12}^2 \exp(-jkl_{CB}) \longrightarrow S_{12}^P = S_{12}^2 \exp(-jkl_{CB})$

$$\underline{\underline{S}}_P = \begin{bmatrix} S_{11}^2 & S_{12}^2 \exp(-jkl) \\ S_{22}^2 \exp(-jkl) & S_{22}^2 \end{bmatrix}$$

Per la matrice "globale", $\underline{\underline{S}}^{eq}$, basta riapplicare le formule:

$$S_{11}^{eq} = S_{11}^1 + \frac{S_{12}^1 S_{21}^1 S_{12}^2 \exp(-j2kl_{BC})}{1 - S_{22}^1 S_{12}^2 \exp(-j2kl_{BC})} ;$$

$$S_{21}^{eq} = \frac{b_2}{a_1}\Big|$$



$V_{2^+}^+ = V_{2^-}^+$; $b_2^P = a_1^P S_{21}^P + \cancel{a_2^P S_{22}^P}$; $b_2^P = a_1^P S_{21}^P$; $b_2^P = b_2$.

$\hookrightarrow V_{2^+}^+ = V_{A^+}^+ S_{21}^P \longrightarrow V_{A^+}^+ = b_2^1$; $b_2^1 = S_{21}^1 a_1^1 + S_{22}^1 a_2^1$; $b_2^1 = a_1^P$

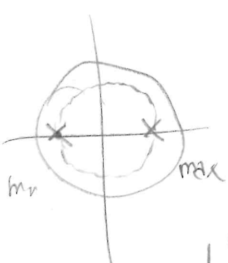
$\hookrightarrow b_2^1 = S_{21}^1 a_1^1 + S_{22}^1 a_2^1 \qquad a_2^1 = \Gamma_A b_2^1 = \Gamma_A a_1^P$

$\hookrightarrow a_1^P [1 - \Gamma_A S_{22}^1] = S_{21}^1 a_1^1$; $\quad \Gamma_A = S_{11}^P = S_{11}^2 \exp(-j2kl)$

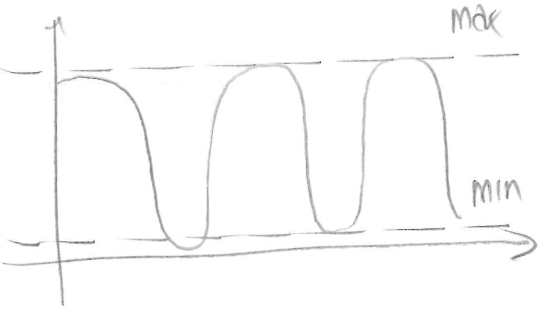
$$a_{eq} = \frac{b_2}{S_{21}^P} \longrightarrow \frac{S_{21}^1 S_{21}^P}{1 - \Gamma_A S_{22}^1} = \frac{S_{21}^1 S_{21}^2 \exp(-jkl)}{1 - S_{11}^2 S_{22}^1 \exp(-j2kl)}$$

Nota: $|S_{21}| = \dfrac{|S_{21}^1||S_{21}^2|}{|1 - S_{22}^1 S_{11}^2 \exp(-j2\beta l)|}$ Al den si ha "$1 - \Gamma_{eq}$": il prodotto dei due S_{ii} è un coeff. di riflessione.

Questo den è pensabile in termini di un <u>coeff. di</u> trasmissione, e noi lo vorremmo idealmente costante al variare di l, in modo da avere trasmissione costante in ogni punto.



$|1 - S_{22}^1 S_{11}^2 \exp(-j2\beta l)|$ — max, $1 + |S_{11}^2||S_{22}^1|$; min, $1 - |S_{11}^2||S_{22}^1|$



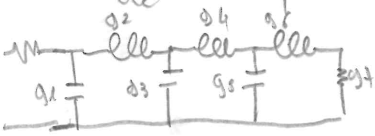
L'ampiezza delle osc. dipende da S_{22}^1 e S_{11}^2, dobbiamo ridurre le oscillazioni, ma dunque <u>adattare</u> S_{22}^1 e S_{11}^2, ossia <u>adattare</u> i blocchi.

Filtri a microonde

Esempietto: $P_{LR}|_{dB} \geq 30\,dB$, $\frac{\omega}{\omega_c} = 1{,}2$, che si fa? Dal graf., sarebbe $N \geq 9$.
ripple 0,5 dB

Step-impedance: dato passa basso $f_c = 2{,}5\,GHz$, @ 4 GHz $P_{LR} \geq 20\,dB$, $Z_0 = 50\,\Omega$, $Z_{0h} = 150\,\Omega$, $Z_{0l} = 10\,\Omega$, massima piattezza in banda.

$\Omega = \frac{\omega}{\omega_c} = 1{,}6$; faccio N=6 per star largo. Progetto a π!



$g_1 = g_6 = 0{,}5176$
$g_2 = g_5 = 1{,}4142$
$g_3 = g_4 = 1{,}9318$
$g_7 = 1.$

$R_L = 50\,\Omega$;

Poi:

$$\begin{cases} Z_h \vartheta_n = g_n R_0 ; \\ Y_l \vartheta_m = g_m Y_0 \end{cases}$$

[βl] = rad

$\vartheta_1 = 5{,}9°$ $\qquad \vartheta_4 = 36{,}9°$

$\vartheta_2 = 27°$ $\qquad \vartheta_5 = 16{,}21°$

$\vartheta_3 = 22{,}1°$ $\qquad \vartheta_6 = 0{,}885°$

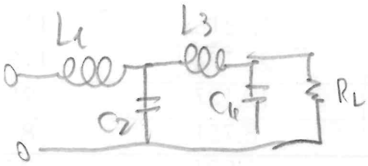
$\vartheta_i = \beta_i l_i$; $\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}\sqrt{\varepsilon_{eff}}$;

Progetto passa-basso con $f_c = 2\,GHz$, $R_0 = 50\,\Omega$, $P_{LR}|_{dB} \geq 15\,dB$, @ $f = 3\,GHz$.
Trovare i filtri prototipo con Butterworth e con Chebishev 0,5 dB.

Li faccio a "T"

Chebishev:

$\Omega = 0{,}5 \Rightarrow N \geq 4$



$R_L = 99{,}21\,\Omega$;

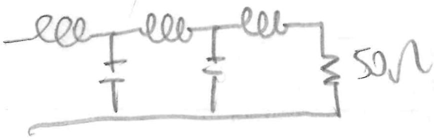
$g_1 = 1{,}6703$
$g_2 = 1{,}1926$
$g_3 = 2{,}3661$
$g_4 = 0{,}8419$

Usando le trasformazioni: $\quad L_1 = 6{,}646\,nH \quad L_3 = 9{,}414\,nH$
$\qquad C_2 = 1{,}9\,pF \quad C_4 = 1{,}34\,pF$

Ricorda: ω_c, NON f_c.

Butterworth:

$N \geq 5$; a "T":



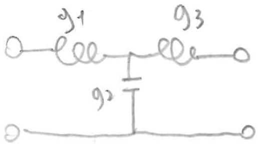
$g_1 = g_5 = 0{,}618$
$g_2 = g_4 = 1{,}618$
$g_3 = 2$

$L_1 = 2{,}5\,nH \qquad C_2 = 2{,}575\,pF$
$L_3 = 7{,}6\,nH \qquad C_4 = $ "
$L_5 = 2{,}5\,nH$

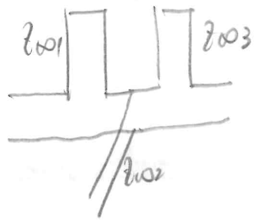
Esercizio

Progettare un passa-basso Chebishov ripple 3 dB tale che $f_c = 4\,GHz$, $R_0 = 50\,\Omega$, $N = 3$ (per ipotesi), $d = \left(\frac{1}{16}\right)''$, $\varepsilon_r = 1,2$

Allora: il filtro "a T" si presta meglio a Kuroda in questo caso.

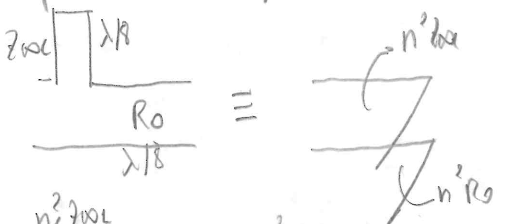


$\Longrightarrow$ $g_1 = g_3 = 3,3487$; $g_2 = 0,7117$ $\longrightarrow$

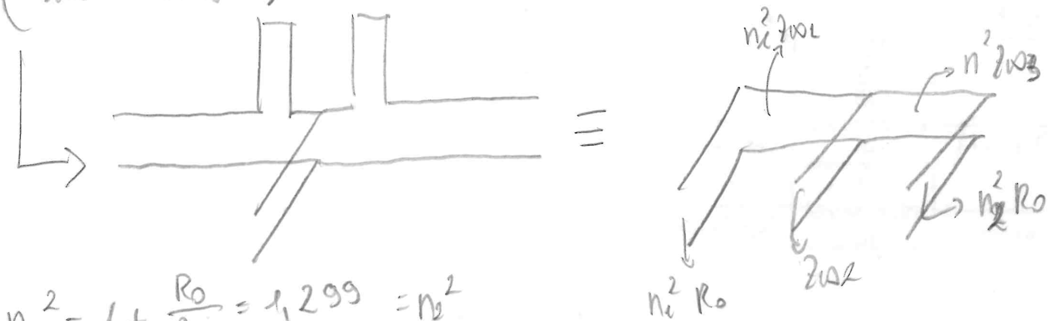


dove $Z_{001} = R_0 g_1$; $Z_{002} = \frac{R_0}{g_2}$; $Z_{003} = R_0 g_3$;

$\begin{cases} Z_{001} = 167,4\,\Omega \\ Z_{002} = 70,25\,\Omega \\ Z_{003} = 167,4\,\Omega \end{cases}$ Ricordo la identità di Kuroda:



$n^2 = 1 + \frac{R_0}{Z_{0L}}$



$n_1^2 = 1 + \frac{R_0}{Z_{00L}} = 1,299 = n_2^2$

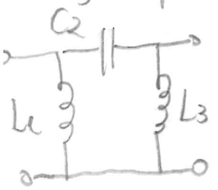
$\begin{cases} n^2 R_0 = 64,93\,\Omega ; \\ n^2 Z_{00L} = 217,4\,\Omega \\ Z_{002} = 70,25\,\Omega \end{cases}$ $d = h = \left[\frac{1}{16}\right] \cdot 2,54 \cdot \frac{1}{100} = 1,588\,mm$

dai grafici:

$\left.\frac{W}{h}\right|_{n^2 R_0} \simeq 3,2$; $\left.\frac{W}{h}\right|_{n^2 Z_{00L}} \simeq 0,22$; $\left.\frac{W}{h}\right|_{Z_{002}} \simeq 2,5$

Esercizio: progettare filtro passa-alto Butterworth, $N = 3$, $f_c = 4\,GHz$, $R_0 = 50\,\Omega$, $K = R_0$, $\varepsilon = 2,54$, $d = \left(\frac{1}{8}\right)''$

Scelgo "per furbizia" filtro a π:



$g_1 = 1$; $g_2 = 2$; $g_3 = 1$; $g_4 = 1$.

$L_n = \frac{R_0}{\omega_0 g_n}$; $\longrightarrow Z_{00L} = Z_{003} = \omega_0 \frac{R_0}{\omega_0 g_n} = \frac{R_0}{g_1}$

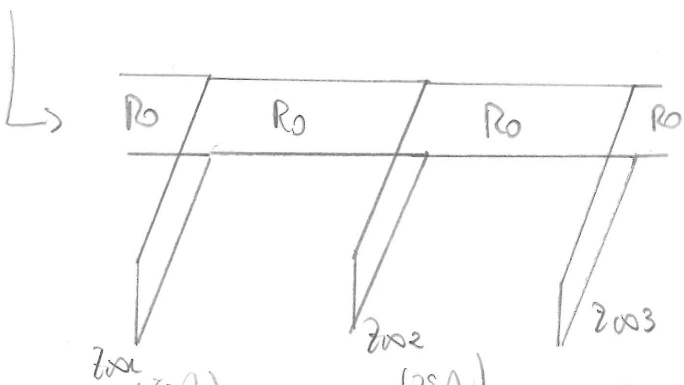
$C_n = \frac{1}{\omega_0 R_0 g_n}$; Ho che (per la teoria dell'invertitore):

$\frac{1}{j\omega C_n} = \frac{1}{j\omega L_p} K^2$;

$\hookrightarrow L_p = C_n K^2 = R_0^2 \frac{1}{\omega_0 R_0 g_n}$;

$Z_{002} = \omega_0 \frac{R_0}{\omega_0 g_n} = \frac{R_0}{g_n} = 25\,\Omega$

$l = \frac{\lambda_g}{8} = \frac{\lambda}{8\sqrt{\varepsilon_{eff}}}$



$(50\,\Omega)$ $\left.\frac{W}{h}\right|_1 = 2,9$; $\sqrt{\varepsilon_{eff}} = 1,45$

$(25\,\Omega)$ $\left.\frac{W}{h}\right|_2 = 7$; $\sqrt{\varepsilon_{eff}} = 1,5$

$\left.\frac{\lambda}{8}\right|_{mm} = 3,175\,mm$;

$W_1 = 9,208\,mm$

$W_2 = 22,23\,mm$

Progettare un risonatore serie in microstriscia usando $Z_0 = 50\,\Omega$, $\varepsilon_r = 2{,}1$, $h = 0{,}159$ cm, $\tan\delta = 0{,}001$, $\sigma = 5{,}813 \times 10^7 \frac{S}{m}$, $f_0 = 5$ GHz

Usando il formulario, con $Z_0 = 50\,\Omega$, $\varepsilon_r = 2{,}1$, si ricava:

$\frac{W}{h} = 3{,}173$; $\varepsilon_{eff} = 1{,}802$; $\sqrt{\varepsilon_{eff}} = 1{,}342$

Si ricorda che $\alpha = \alpha_d + \alpha_c$;

$\alpha_d = 0{,}05991$

$R_s = 18{,}43 \cdot 10^{-3}\,\Omega$; $\rightarrow \alpha_c = 0{,}07305$

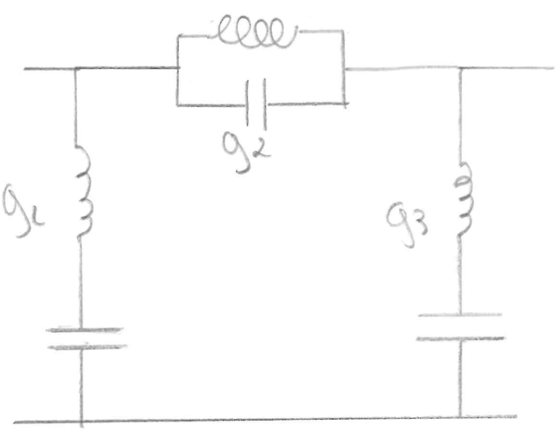
$\alpha = \alpha_c + \alpha_d = 132{,}7 \times 10^{-3}$; $\beta = 140{,}6$; $l = \frac{\lambda_g}{2} = \frac{c}{f_0\sqrt{\varepsilon_{eff}}} = 11{,}18$ mm

Ricordando le formule:

$R = Z_0 \alpha l$; $Q = \frac{\beta}{2\alpha}$;

$R = 0{,}072\,\Omega$; $Q = 540{,}1$

Progettare un filtro rigetta-banda: Chebishev con $N=3$, ripple di 0,5 dB, $R_0 = 50\,\Omega$, $f_0 = 2$ GHz, $\Delta = 0{,}15$



$g_1 = g_3 = 1{,}5963$;

$g_2 = 1{,}097$

Per il ris. parallelo centrale si deve usare l'invertitore di impedenza, facendo riferimento a questo procedimento: alla fine di tutto il procedimento si avrà un ris. serie, dunque si consideri la formula del ris. serie:

$L = \frac{\pi}{4}\frac{Z_0}{\omega_0} \rightarrow Z_{0A} = \frac{4\omega_0}{\pi} L_n$ $\quad$ L_n fa riferimento a $K^2 C_n$, dal momento che uso l'invertitore di impedenza; se L_n appartiene al ris. serie, C_n appartiene al ris. parallelo di partenza. Uso dunque come C_n quella del rigetta-banda, relativo al risonatore parallelo:

$C_n = \frac{1}{g_n \Delta R_0 \omega_0}$;

$L \rightarrow Z_{0n} = \frac{4\omega_0}{\pi} K^2 \frac{1}{g_n \Delta R_0 \omega_0} = \frac{4 R_0}{\Delta g_n \pi}$ $\rightarrow$ Questa è la formula finale!

Calcolare la matrice $\underline{\underline{S}}$ con i seguenti dati: ⑥

$$\underline{\underline{S}}^c = \begin{bmatrix} 0 & S_{12}^c & 0 \\ 0 & 0 & S_{23}^c \\ S_{31}^c & 0 & S_{33}^c \end{bmatrix}$$

$|S_{31}^c| = |S_{23}^c| = |S_{12}^c| = -0{,}5 \text{ dB}$;

$|S_{33}| = -18 \text{ dB}$; $\angle S_{33} = 0$;

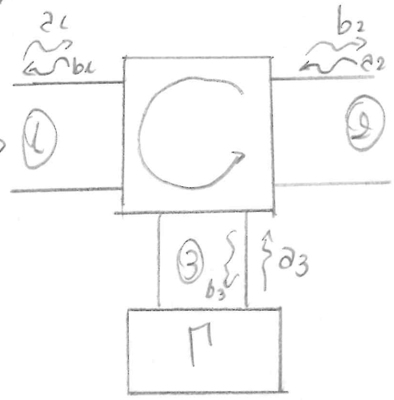
$\angle S_{31} = \angle S_{23} = \angle S_{12} = \frac{\pi}{3}$

$\Gamma = 0{,}5$; $P_{inc} = 1 \text{ mW}$

Disegnare il circuito, e calcolare P_2.

Si tratta di un circolatore con perdite. Dalla matrice, e dalle relative equazioni, si può evincere che:

$$\begin{cases} b_1 = S_{12}^c a_2 \\ b_2 = S_{23}^c a_3 \\ b_3 = S_{31}^c a_1 + S_{33}^c a_3 \end{cases}$$



Risolviamo:

$b_2 = S_{23}^c a_3$; $a_3 = \Gamma b_3$;

$$b_3 = S_{31}^c a_1 + S_{33}^c \Gamma b_3 \rightarrow b_3 [1 - \Gamma S_{33}^c] = S_{31}^c a_1 \rightarrow b_3 = \frac{S_{31}^c a_1}{1 - \Gamma S_{33}^c}; \; a_3 = \Gamma b_3 = \frac{\Gamma S_{31}^c a_1}{1 - \Gamma S_{33}^c};$$

$$b_2 = \frac{S_{23}^c \Gamma S_{31}^c}{1 - \Gamma S_{33}^c} a_1 \rightarrow S_{21} = \frac{S_{23}^c \Gamma S_{31}^c}{1 - \Gamma S_{33}^c}$$

$$\frac{P_{out}}{P_{inc}} = \frac{\frac{1}{2}|b_2|^2}{\frac{1}{2}|a_1|^2} = \frac{|S_{23}^c \Gamma S_{31}^c|^2}{|1 - \Gamma S_{33}^c|^2} = |S_{21}|^2;$$

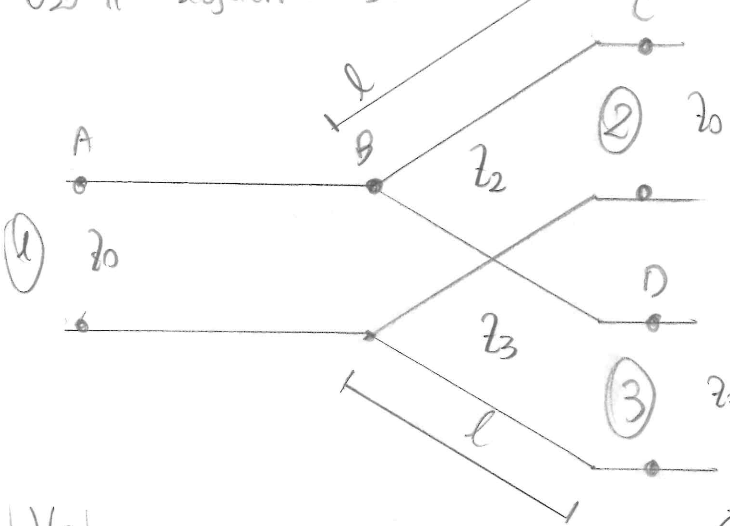
Tiriamo fuori i numeri:

$|S_{31}| = 0{,}9441$; $|S_{33}| = 0{,}1259$;

$$\frac{P_{out}}{P_{inc}} = 0{,}226.$$

Progettare un divisore di potenza a T tale che $\frac{V_2}{V_3} = -0{,}23$, $Z_0 = 50\,\Omega$ (7)

Uso il seguente schema: $l = \frac{\lambda}{4}$



Dalla teoria, è noto che:

$$V_C = V_{C^-}^+ \left[1 + \Gamma_{C^-}\right] = V_{B^+C}^+ \left[1 + \Gamma_{C^-}\right] \exp(-jKl) = V_{B^-}^+ \left[1+\Gamma_{C^-}\right] \frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}} = V_{B^-}^+ \zeta_{C^-} (-j) \left[1+\Gamma_{B^-}\right];$$

$$V_D = V_{D^-}^+ \left[1+\Gamma_{D^-}\right] = V_{B^+D}^+ \left[1+\Gamma_{D^-}\right] \exp(-jKl) = V_{B^-}^+ \left[1+\Gamma_{D^-}\right] \frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}} \exp(-jKl) = V_{B^-}^+ \cdot \zeta_{D^-} (-j) \left[1+\Gamma_{B^-}\right];$$

$$\left|\frac{V_C}{V_D}\right| = 0{,}23 \rightarrow \frac{\zeta_{C^-}}{\zeta_{D^-}} = 0{,}23; \quad \zeta_{C^-} = \frac{Z_0}{Z_2}; \quad \zeta_{D^-} = \frac{Z_0}{Z_3}; \rightarrow \frac{Z_3}{Z_2} = 0{,}23;$$

Questa è una condizione; per avere adattamento alla porta 1, inoltre, dovremo imporre che il parallelo delle resistenze Z_{B^+C} e Z_{B^+D} sia Z_0; dunque:

$$\frac{1}{Z_0} = \frac{1}{\frac{Z_2^2}{Z_0}} + \frac{1}{\frac{Z_3^2}{Z_0}} = \frac{Z_0}{Z_2^2} + \frac{Z_0}{Z_3^2};$$ con la TI89;

$Z_3 = 51{,}31\,\Omega$; $Z_2 = 223{,}1\,\Omega$.

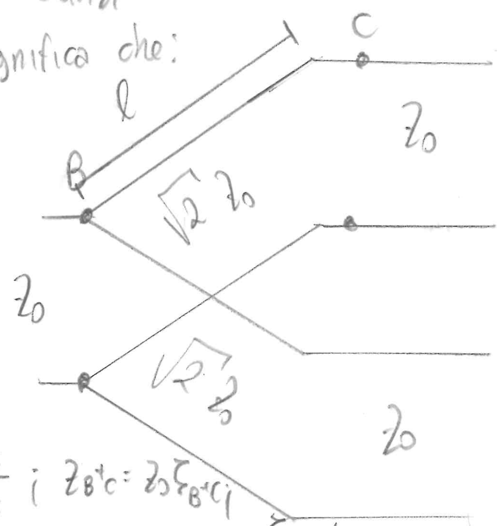
Dato un divisore di potenza Wilkinson bilanciato, $Z_0 = 50\,\Omega$, $f = 1\,GHz$; calcolare S_{11} e S_{21} @ $0{,}5\,GHz$, considerando a $1\,GHz$ tutto adattato.

Ragionamento preliminare: dire che $\frac{f}{f_0} = 0{,}5$, significa che:

$K_0 = \frac{2\pi f_0}{c} = 2\frac{2\pi f}{c} = 2K$

Dunque, l'argomento dell'esponenziale si dimezza.

Si ottiene, in sostanza, che $l = \frac{l_0}{2} = \frac{\lambda}{8}$



Dai conti:

$$\Gamma_{C^-} = \frac{Z_0 - \sqrt{2}\,Z_0}{Z_0 + \sqrt{2}\,Z_0} \approx -0{,}1716;$$

$$\Gamma_{B^+C} = \Gamma_C \cdot \exp(-j2Kl) = -j\,\Gamma_C = j\,0{,}1716; \quad \zeta_{B^+C} = \frac{1+\Gamma_{B^+C}}{1-\Gamma_{B^+C}}; \quad Z_{B^+C} = Z_0\zeta_{B^+C};$$

$$Z_{B^-} = \left(Z_0\zeta_{B^+C}\right) \oplus \left(Z_0\zeta_{B^+D}\right) = \left(Z_0\zeta_{B^+C}\right) \oplus \left(Z_0\zeta_{B^+C}\right) = 0{,}707\exp(j\,0{,}3398); \quad \Gamma_{B^-} = \frac{\zeta_{B^-}-1}{\zeta_{B^-}+1} \approx 0{,}2425\exp(j\,2{,}386);$$

Con lo stesso circuito si può calcolare S_{21}:

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=a_3=0} = \frac{V_{C^+}^+}{V_{A^-}^+}; \quad V_{C^+}^+ = V_{C^-}^+\left[1+\Gamma_{C^-}\right] = V_{B^+C}^+ \exp(-jKl)\left[1+\Gamma_{C^-}\right] = V_{B^-}^+ \frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+C}}\left[1+\Gamma_{C^-}\right]\exp(-jKl)$$

Esercizio - filtro di diramazione

Dato un sistema con $|S_{11}| = 0{,}95$, $|S_{21}|_{dB} = -0{,}1$ dB, dato $s(f)$ in cui:

$s(f) = s_1(f_1) + s_2(f_2) + s_3(f_3) + s_4(f_4)$

Qual è il segnale più attenuato?

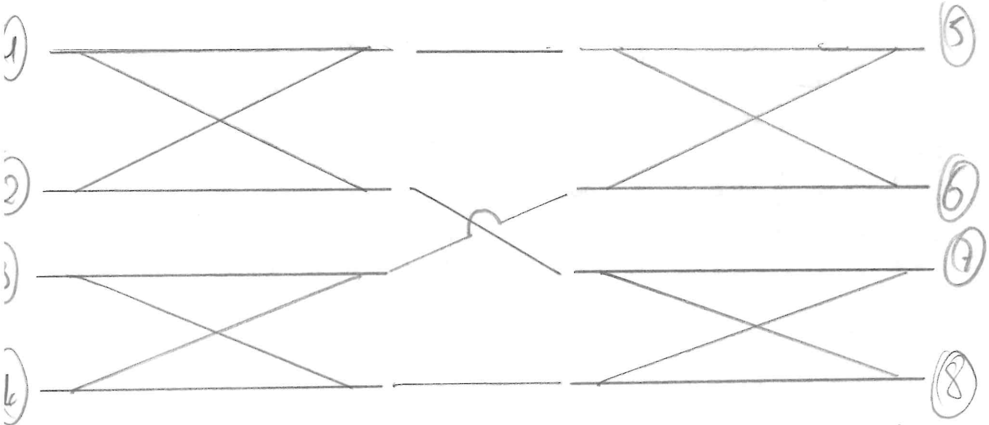
Ricordo che i coeff. scattering son lineari; dunque, $|S_{21}| = 10^{\frac{-0{,}1}{20}} = 0{,}9886$;

Al primo colpo rifletto "$s_2 + s_3 + s_4$" e trasmetto s_1; al secondo trasmetto s_2 e rifletto $s_3 + s_4$; al terzo trasmetto s_3 e rifletto s_4.

Per s_3: $s_{3out} = 0{,}95^3 s_{3in} \approx 0{,}857 s_{3in}$; s_4: $s_{4out} = 0{,}95^2 s_{4in} \cdot 0{,}9886 \approx 0{,}892 s_{4in}$

Esercizio: rete di distribuzione del segnale.

Dati 4 accoppiatori uguali:



$$P_5 = P_1 [1-\beta^2]^2 + P_2 \beta^2 [1-\beta^2] + P_3 [1-\beta^2] \beta^2 + P_4 (\beta^2)^2$$

Esercizio

Dato un segnale da distribuire a diversi utenti, $P_{in} = 10$ mW, $\beta^2 = \frac{1}{2}$, $P_{out\,min} = 10\ \mu W$, $l = 3$m tra un utente e un altro, $\alpha_{dB} = 0{,}05$ dB/m, quanti utenti posso servire?

$$e^{-2\alpha l} = 10^{\frac{-0{,}05 \cdot 3}{10}} = 0{,}966;$$

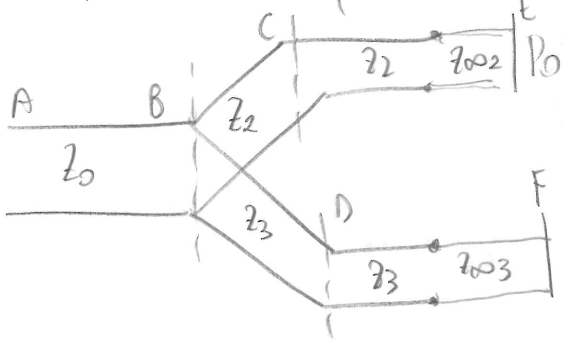
Il segnale si propaga per $n-1$ volte, e la n-esima si accoppia; supponendo tutto adattato, non avremo attenuazione di disadattamento, dunque:

$$P_{out} = P_{in} \cdot (1-\beta^2)^{n-1} \cdot \beta^2 \cdot (e^{-2\alpha l})^{n-1}$$; fissate P_{in} e P_{out}, si inverte (con la calcolatrice)

$\rightarrow n = 9{,}54 \longrightarrow \boxed{n = 9}$

Alcuni esercizi dal Pozar

7.5) Data $R_0 = 30\,\Omega$, progettare un divisore T-junction con rapporto di potenza 3:1; usare/progettare adattatori $\lambda/4$; Valutare $\underline{\underline{S}}$.



Potenza 3:1;

$P_C = \frac{1}{2}|V_C|^2 Y_2$;

$P_D = \frac{1}{2}|V_D|^2 Y_3$;

$\longrightarrow \frac{P_C}{P_D} = 3 = \frac{Y_2}{Y_3}$;

$\longrightarrow Y_2 = 3Y_3$;

$\hookrightarrow Y_0 = Y_2 + Y_3 = 4Y_3$; $\longrightarrow Y_3 = 8{,}\bar{3}\ mS$; $Z_3 = 120\,\Omega$; $Z_2 = 40\,\Omega$;

Gli adattatori $\lambda/4$ varranno:

$Z_{002} = \sqrt{30 \times 40} = 34{,}64\,\Omega$;

$Z_{003} = \sqrt{30 \times 120} = 60\,\Omega$

Si vuol dunque calcolare $\underline{\underline{S}}$; considero tutto adattato a $Z_0 = 30\,\Omega$; ciò ci porta ad avere:

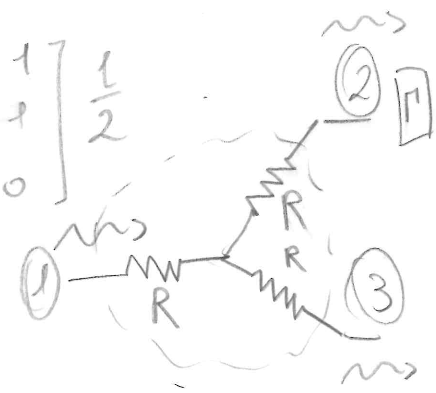
$\Gamma_A = 0$; $\longrightarrow S_{11} = 0$; S_{21}: $S_{21} = \frac{b_2}{a_1}\Big|_{a_2 = a_3 = 0}$; $b_2 = \frac{V_E^{+}}{\sqrt{Z_0}}$; $a_1 = \frac{V_A^{+}}{\sqrt{Z_0}}$; $\longrightarrow S_{21} = \frac{V_E^{+}}{V_A^{+}}$

$\hookrightarrow V_{E^+}^{+} = V_{E^-}^{+} \frac{1+\Gamma_E^-}{1+\Gamma_{E^+}} = V_{C^+}^{+} \exp(-jkl)(1+\Gamma_E^-) = V_C^{+} \frac{1+\Gamma_{C^-}}{1+\Gamma_{C^+}}(1+\Gamma_E^-)\exp(-jkl) =$

$= -j\,\zeta_E\,(-j)\,V_{B^+C}^{+} = -\zeta_{E^-} \frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}} V_{B^-}^{+} = -\zeta_{E^-} V_{B^-}^{+}$

7.7) Ricordando che:

$R = \frac{Z_0}{3} = 33{,}\bar{3}\,\Omega$; e che: $\underline{\underline{S}} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \frac{1}{2}$



Vogliam calcolare in dB quanta potenza "cambia" quando 2 è connesso a un adattatore o a $\Gamma = 0{,}3$

$b_3 = S_{31}a_1 + S_{32}a_2$; HA:

se $a_2 = 0$, $\frac{b_3}{a_1} = S_{31}$;

se $a_2 \neq 0$, $a_2 = \Gamma b_2 = \Gamma[S_{21}a_1 + S_{23}a_3]$;

$\hookrightarrow Ris = \frac{|S_{31}|^2 |a_1|^2}{|S_{31}a_1 + S_{32}\Gamma S_{21}a_1|^2} = \frac{|S_{31}|^2}{|S_{31} + \Gamma S_{32}S_{21}|^2}\Bigg|_{dB} =$

$S_{31} = S_{32} = S_{21} = \frac{1}{2}$;

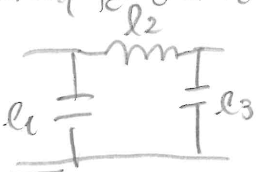
$\Gamma = \frac{3}{10}$;

$\longrightarrow \approx -1{,}21\ dB$

8.13) But with shunt stubs. $g_1 = g_3 = 1$; $g_2 = 2$; $g_4 = 1$.

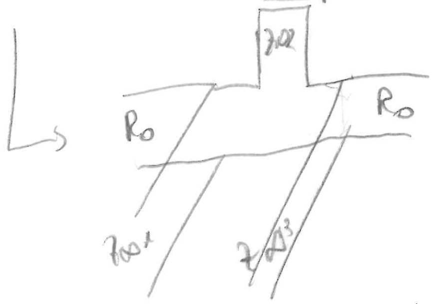
$N = 3$, Butterworth, $f_c = 6\,GHz$;

Uso a π:



$j\omega l_i \to j\frac{\omega}{\omega_0} c_i = j\frac{\omega}{\omega_0}\frac{g_n}{R_0} = j\omega C_n$, $C_n = \frac{g_n}{R_0 \omega_0}$;

$j\omega l_i \to j\frac{\omega}{\omega_0} g_m R_0 = j\omega L$, $L = \frac{g_m R_0}{\omega_0}$ $\quad Z_{0C} = \frac{g_n}{R_0}$;



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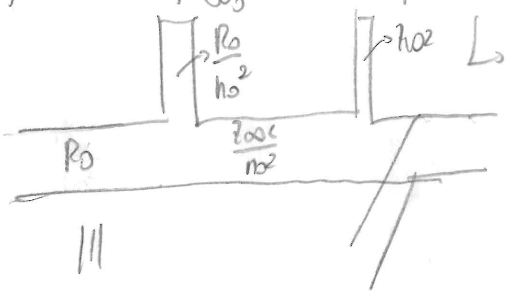
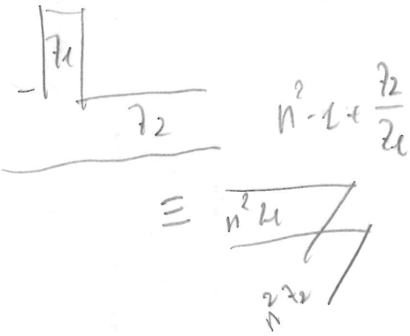


$L \to Z_{0L} = \frac{R_0}{\omega_0}$



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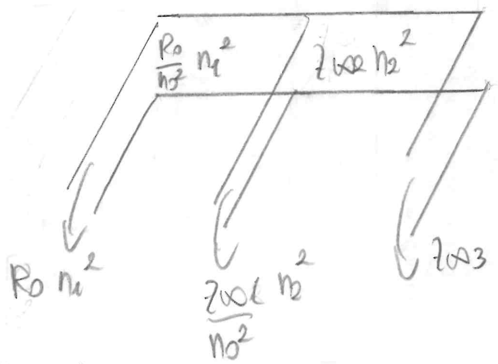
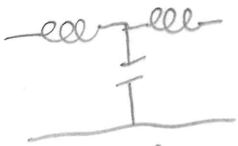


$n_0^2 = \frac{Z_{0L}}{R_0} + 1$

$n_1^2 = 1 + \frac{R_0}{\frac{R_0}{n_0^2}} = 1 + n_0^2$

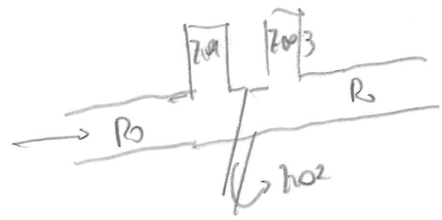
$n_2^2 = \frac{\frac{Z_{0L}}{n_0^2}}{Z_{02}} = \frac{Z_{0L}}{n_0^2 Z_{02}} + 1$

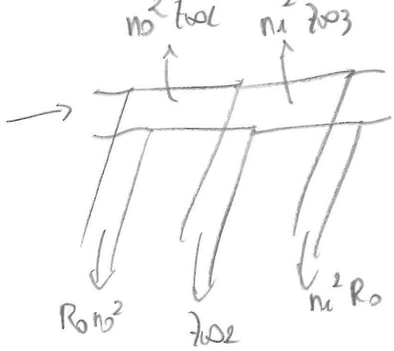
Come veniva a "T"? Vediamo:



$Z_{0L} = g_m R_0$

$Z_{0C} = \frac{g_m}{R_0}$





$n_0^2 = \frac{R_0}{Z_{0L}} + 1$;

$n_1^2 = 1 + \frac{R_0}{Z_{03}}$;

(11)

8.17) $f_c = 3GHz$, Chebi 0,5dB $N=5$; $Z_l = 15\Omega$, $Z_h = 120\Omega$;

$g_1 = g_5 = 1,7058$; $g_2 = g_4 = 1,2296$; $g_3 = 2,5408$; $g_5 = L$; $R_0 = 50\Omega$

stepped impedance: $\begin{cases} Z_h \vartheta_{Ln} = R_0 g_n ; \\ Y_l \vartheta_n = Y_0 g_n ; \end{cases}$ $d = 0,079$ cm $= 0,79$ mm ; $\epsilon_r = 4,2$; $\beta = \frac{2\pi f}{c} \sqrt{\epsilon_{eff}}$

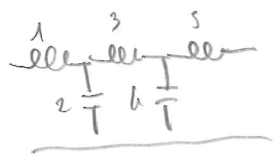
Dai grafici:

		ϵ_{eff}	β	W (mm)
15	10 (13)	3,756	121,8	10,27
120	0,3 (0,19)	2,8	105,1	150 μm

$\vartheta = \beta l$;

$l_L = \frac{R_0 g_n}{Z_h \beta_L}$

$l_c = \frac{Y_0 g_n}{Y_l \beta_c} = \frac{Z_l g_n}{Z_0 \beta_c}$



$\longrightarrow$ $l_1 = 5,8$ mm; $l_2 = 3,5$ mm; $l_3 = 8,7$ mm; $l_4 = 3,5$ mm; $l_5 = 5,8$ mm

8.20) $N = 4$; bandstop; open-circ quarter-wave; $f_0 = 3GHz$, $\Delta = 0,15$, $R_0 = 40\Omega$

$C_n = \frac{g_n}{R_0}$; $\frac{1}{j\omega C_n} \Rightarrow \frac{-1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$

$\omega \to -\Delta \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1}$

$\hookrightarrow \frac{1}{\Delta}\frac{\omega}{j\omega_0 C_n} - \frac{1}{\Delta}\frac{1}{jC_n}\frac{\omega_0}{\omega} \to \frac{1}{j\omega C_n}$, $\frac{1}{C_n} = \frac{\omega_0 R_0}{\Delta g_n}$

$\hookrightarrow j\omega L_n \to L_n = \frac{R_0}{\Delta\omega_0 g_n}$ $\quad C_n = \frac{\Delta g_m}{\omega_0 R_0}$

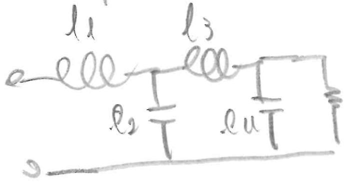
$\hookrightarrow Z_0 = \frac{4}{\pi} \omega_0 L_n =$

$= \frac{4}{\pi} \omega_0 \frac{R_0}{\Delta \omega_0 g_n} = \boxed{\frac{4 R_0}{\pi \Delta g_n}}$

Progetto concluso: tutto noto!

8.14 | Low-pass, $N=4$, Butterworth, Kuroda ; $f_c = 8$ GHz. $Z_0 = 50\,\Omega$. (12)

Uso questo prototipo:



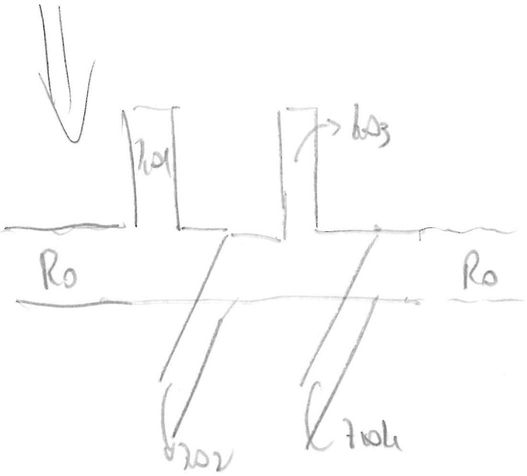
$g_1 = g_4 = 0{,}7654$;
$g_2 = g_3 = 1{,}8478$

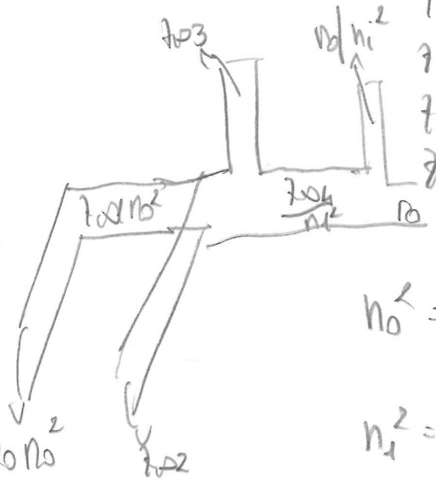
Dalle trasf. di IMPEDENZA: $\begin{cases} l_i = g_i R_0 \\ c_i = \dfrac{g_i}{R_0} \end{cases}$;

Freq.: $\omega \to \dfrac{\omega}{\omega_0}$;

$j\omega l_i \to j\dfrac{\omega}{\omega_0} l_i = j\omega L_i$, $L_i = \dfrac{g_m R_0}{\omega_0}$; $C_i = \dfrac{g_m}{R_0 \omega_0}$

Richards: $\longrightarrow$ $Z_{0L} = g_m R_0$; $Z_{0C} = \dfrac{1}{\frac{g_m}{R_0 \omega_0}\omega_0} = \dfrac{R_0}{g_m}$;





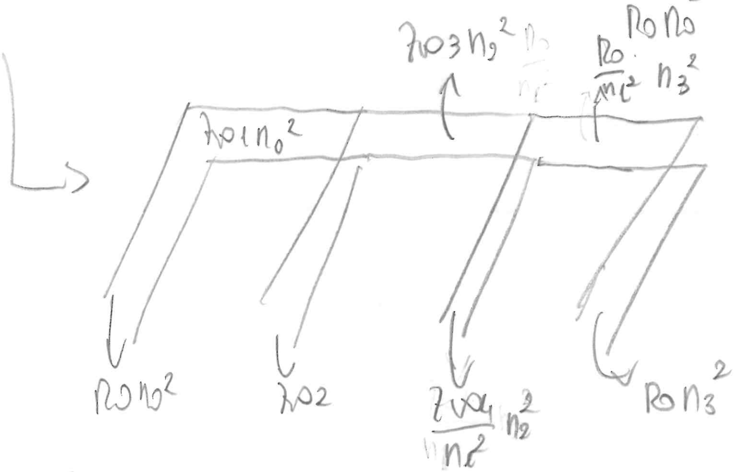
$Z_{01} = 38{,}27\,\Omega$
$Z_{02} = 27{,}06\,\Omega$
$Z_{03} = 92{,}39\,\Omega$
$Z_{04} = 65{,}33\,\Omega$

$n_0^2 = \dfrac{R_0}{Z_{01}} + 1 = 2{,}307$

$n_1^2 = \dfrac{Z_{04}}{R_0} + 1 = 2{,}307$

$n_2^2 = \dfrac{\frac{Z_{04}}{n_1^2}}{Z_{03}} + 1 = \dfrac{Z_{04}}{Z_{03} n_1^2} + 1 = 1{,}307$

$n_3^2 = \dfrac{R_0}{\frac{R_0}{n_1^2}} + 1 = n_1^2 + 1 = 3{,}307$

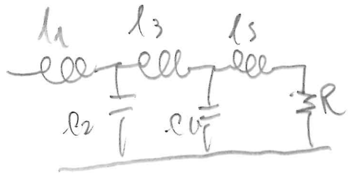


$R_0 n_0^2 = 115{,}3\,\Omega$
$Z_{01} n_0^2 = 88{,}27\,\Omega$
$Z_{02} = 27{,}06\,\Omega$
$Z_{03} n_2^2 = 120{,}7\,\Omega$
$\dfrac{Z_{04}}{n_1^2} n_2^2 = 37$

$\dfrac{R_0}{n_1^2} n_3^2 =$
$R_0 n_3^2 =$

8.18) $f_c = 2GHz$; $R_0 = 50\Omega$; $N = 5$ Butterworth $Z_l = 10\Omega$, $Z_h = 150\Omega$. (13)

$g_1 = g_5 = 0,6180$; $g_2 = g_4 = 1,6180$; $g_3 = 2$; $g_6 = 1$; $\varepsilon = 2,54$ (MIA ipotesi) $n = \left(\frac{1}{8}\right)^{h}$



Stepped impedance: $\begin{cases} Z_h \vartheta_m = Z_0 g_m \\ Y_l \vartheta_m = Y_0 g_m \end{cases}$ $\vartheta = \beta l$; $[\vartheta] = rad.$

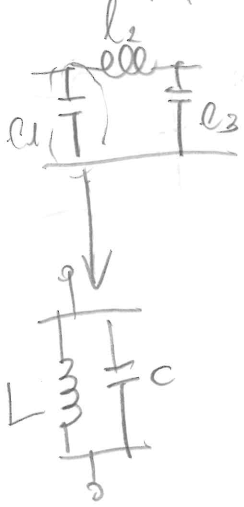
Dai grafici:

$\frac{W}{h}\Big|_{10\Omega} \cong 10$; calcoli: 21. ; $\varepsilon_{eff} = 2,384$; $\beta = \frac{2\pi f}{c} \cdot \sqrt{\varepsilon_{eff}} = 64,68$; $l = \frac{g_m Z_l}{Z_0 \beta}$

$\frac{W}{h}\Big|_{150\Omega} = 0,26$; calcoli: 0,2558 ; $\varepsilon_{eff} = 2,093$ $\beta = \frac{2\pi f}{c} \cdot \sqrt{\varepsilon_{eff}} = 60,6$; $l = \frac{g_m Z_0}{Z_h \beta}$

8.21) Band-pass 0,5dB-equal response, $\omega_0 = 3GHz$, $\Delta = 0,2$, $Z_0 = 100\Omega$;

$\varepsilon_r = 4,2$, $d = 0,079$, $(\tan\delta = 0,02)$ $N = 3$; $g_1 = g_3 = 1,5963$; $g_2 = 1,0967$



Ricordo che: $c_i \Rightarrow j\omega c_i \rightarrow j\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) =$

$= \left(j\frac{1}{\Delta}\frac{\omega}{\omega_0}c\right) - j\frac{1}{\Delta}\frac{\omega_0}{\omega}c$;

$\| j\omega C$; $C = \frac{1}{\Delta\omega_0}c = \boxed{\frac{g_m}{\Delta\,\omega_0 R_0}}$;

dove $C = \frac{\pi}{4\omega_0 R_0}$; $Z_{0i} = \frac{\pi}{4\omega_0 C} = \frac{\pi}{4\omega_0 \frac{g_m}{\Delta\omega_0 R_0}} = \frac{\pi\Delta R_0}{4 g_m}$

$Z_{01} = Z_{03} = 9,84\,\Omega$ Ricorda: sono ros. piccoli,

$Z_{02} = 8,238\,\Omega$ ma ci sta.

$Z_{03} =$

Esercitazioni a casa

Esercitazione 1

1) $Z_{0_{01}} = 30\,\Omega$; $Z_{0_{02}} = 60\,\Omega$; $Z_P = Z_{0_{02}}$; $\overline{AB} = \lambda/3$; $\overline{BC} = 3\lambda/8$

$S_{11} = \Gamma_A = \frac{b_1}{a_1}\Big|_{a_2=0}$; $Z_{B^+} = Z_{0_{02}}$; $Z_P = Z_{0_{02}} \rightarrow Z_{B^-} = Z_{0_{02}} \| Z_{0_{02}} = Z_{0_{01}}$

$\hookrightarrow \Gamma_{A^-} = 0$; $\rightarrow S_{11} = 0$;

$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0} = \sqrt{\frac{Z_{0_{01}}}{Z_{0_{02}}}}\,\frac{V_{C^+}^+}{V_{A^-}^+} \Rightarrow V_{C^+}^+ = V_{C^-}^+ = V_{B^+}^+ \exp(-jkl_2) = V_{B^-}^+ \exp(-jkl_2) = V_{A^+}^+ \exp(-jk(l_1+l_2))$

$S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0} \rightarrow Z_{B^+} = Z_{0_{01}}$; $Z_P = Z_{0_{02}} \rightarrow Z_{B^-} = Z_{0_{01}} \| Z_{0_{02}} = 20\,\Omega$; $\Gamma_{B^-} = \frac{20-60}{20+60} = \boxed{-0{,}5}$;

$\hookrightarrow \Gamma_{C^-} = \Gamma_{B^-} \exp(-2jkl_2)$

2) Calcolo le imp. eq. degli stub: $\overline{AD} = S_1$, $\overline{BE} = S_2$;

$Z_E = jZ_{0_{01}}$; $\Gamma_{E^-} = \frac{jZ_{0_{01}} - Z_{0_{01}}}{jZ_{0_{01}} + Z_{0_{01}}} = \frac{j-1}{j+1}$; $\overline{BE} = \lambda/8 \rightarrow \Gamma_{B^+E} = \frac{j-1}{j+1} \exp\left(-j2\frac{2\pi}{\lambda}\frac{\lambda}{8}\right) =$

$= -j\,\frac{j-1}{j+1} = \frac{j+1}{j+1} = 1$; dato $\Gamma = 1$, $Z_{B^+E} = \infty \rightarrow$ circuito aperto! Come se lo stub non ci fosse!

Per S_1: $Z_{0_{03}} = \frac{Z_0}{2}$; $\Gamma_{D^-} = 1$; $\overline{AD} = 5\lambda/8 \rightarrow \Gamma_{A^+D} = 1\exp\left(-2j\frac{2\pi}{\lambda}\frac{5\lambda}{8}\right) = \exp\left(-j\frac{5\pi}{2}\right) = \exp\left(-j\frac{\pi}{2}\right)$

$= -j$; $\zeta_{A^+D} = \frac{1-j}{1+j}$; $Z_{A^+D} = \frac{1-j}{1+j}\,\frac{Z_0}{3} = \frac{(1-j)^2}{2} = \frac{1+1-2j}{2} = -j\frac{Z_0}{2}$

S_{11}: $\frac{b_1}{a_1}\Big|_{a_2=0} \rightarrow Z_{ri} = Z_0$; $Z_{0_{01}} = 2Z_0$; $\Gamma_{C^-} = \frac{Z_0 - 2Z_0}{Z_0 + 2Z_0} = -\frac{1}{3}$; $\Gamma_{B^+} = -\frac{1}{3}\exp\left(-2jk\frac{3\lambda}{4}\right) =$

$= +\frac{1}{3}$; $Z_{B^+} = \frac{1}{Z_{A^-}}$; $Z_{B^+} = 2Z_0\,\frac{1+\frac{1}{3}}{1-\frac{1}{3}} = 4Z_0$; $Z_{0_{02}} = \frac{Z_0}{3} \rightarrow \boxed{\zeta_{B^-} = 12}$

$\Gamma_{B^-} = 0{,}846$; $\Gamma_{A^+B} = \Gamma_{B^-}\exp\left(-j2k\frac{\lambda}{2}\right) = \Gamma_{B^-}$; $\rightarrow Z_{A^-B} = 4Z_0$; $Z_{A^-} = 0{,}496\exp(-j\,1{,}446)$;

$S_{11} = 0{,}905\exp(-j\,2{,}224)$

$S_{21} = \frac{V_{C^+}^+}{V_{A^-}^+} \Rightarrow V_{C^+}^+ = V_C^+(1+\Gamma_{C^-}) = V_{B^+C}^+(1+\Gamma_{C^-})\exp(-jkl_{BC}) = V_{B^-}^+\,\frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}}(1+\Gamma_{C^-})\exp(-jkl_{BC}) =$

$= V_{A^+B}^+\exp(-jk(l_{AB}+l_{BC}))\,\frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}}(1+\Gamma_{C^-}) = V_{A^-}^+\,\frac{1+\Gamma_{A^-}}{1+\Gamma_{A^+}}\,\frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}}(1+\Gamma_{C^-})\exp\left(-j\frac{2\pi}{\lambda}\left(\frac{\lambda}{2}+\frac{3\lambda}{4}\right)\right)$

($1+\Gamma_{A^-}$: S_{11}; $\frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}}(1+\Gamma_{C^-})$: ζ_{C^-}; $\frac{1}{2}+\frac{3}{4} = \frac{2+3}{4} = \frac{5}{4}$; $= \frac{\pi}{2}$)

$\rightarrow = V_{A^-}^+ \cdot (-j) \cdot \zeta_{C^-} \cdot (1+S_{11}) \Rightarrow S_{21} = 0{,}426\exp(-j\,2{,}583)$

3) $\alpha_{dB} = 0{,}8\,dB/m$; $BC = \frac{11\lambda}{4}$; $\lambda = 1\,m \rightarrow 0{,}8 \times \frac{11}{4} = 2{,}2$; $e^{-2\alpha x} = 0{,}6026$; (2)

$\Gamma_C^- = \frac{5Z_0/2 - 2Z_0}{5Z_0/2 + 2Z_0} = \frac{5/2-2}{5/2+2} = \frac{1/2}{9/2} = \frac{1}{9}$; $\Gamma_{B^+} = \frac{1}{9}\exp(-j2kz) = \frac{1}{9}\exp(-2\alpha z)\exp(-j2\beta z)$;

$\Gamma_{B^+} = 0{,}6026 \cdot \frac{1}{9} \cdot \exp(-j2\beta z) = 0{,}06695 \exp\left(-j2\frac{2\pi}{\lambda}\frac{11\lambda}{4}\right) = 0{,}06695\exp(-j11\pi) = -0{,}06695$;

$\Gamma_{A^+} = 0{,}2793$; $\zeta_{A^+} = 1{,}775$; $V_A = V_g \cdot \frac{1{,}775 Z_0}{(0{,}5+1{,}775)Z_0} = 0{,}780\,V_g = 7{,}8\,V$;

$P_A = \frac{1}{2}\frac{|V_A|^2}{Z_A} = \frac{17{,}15}{Z_0}\,W$;

$$\begin{cases} b_1 = S_{11}a_1 + S_{12}a_2 \\ b_2 = S_{21}a_1 + S_{22}a_2 \end{cases} \longrightarrow b_2 = S_{21}a_1 + S_{22}a_2;\quad a_2 = \Gamma b_2 \longrightarrow b_2\left[1 - S_{22}\Gamma\right] = S_{21}a_1$$

$\hookrightarrow \frac{b_2}{a_1} = \frac{S_{21}}{1-\Gamma S_{22}}$; $\frac{P_B}{P_A} = \frac{|S_{21}|^2}{|1-\Gamma S_{22}|^2}$; $\Gamma = 0{,}272$;

$\frac{P_C}{P_A} = \frac{|S_{21}|^2}{|1-\Gamma_B S_{22}|^2}\;\frac{1-|\Gamma_{B^-}|^2}{1-|\Gamma_{A^+}|^2}\;\frac{1-|\Gamma_{C^-}|^2}{1-|\Gamma_{B^+}|^2}\exp(-2\alpha l) \simeq \frac{6{,}962}{Z_0}$

RIVEDERE

4) $\frac{130-90}{200} = \frac{\lambda}{2}$; $\Gamma_{B^+} = -0{,}233$; $\Gamma_{B^-} = -0{,}44$; $\Gamma_{A^+} = +0{,}44$; $Z_{A^+} = 80 \times$

$P_{max} = \frac{|V_A^+|^2}{2\,200}|\Gamma_A|^2$; $P_A = P_A^+\left(1-|\Gamma_A|^2\right) = \frac{P_{max}}{|\Gamma_A|^2}\left(1-|\Gamma_A|^2\right) = \frac{0{,}2}{|\Gamma_A|^2}\left(1-|\Gamma_A|^2\right) = 0{,}831\,mW$

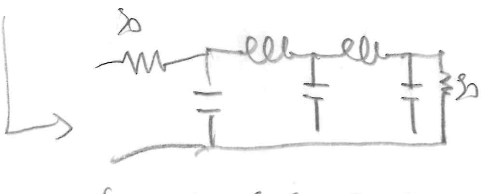
$P_C = P_A \frac{1+|\Gamma_B|^2}{1+|\Gamma_A^+|^2} \cdot \frac{|S_{21}|^2}{|1-\Gamma_C S_{22}|^2} \cdot \frac{1+|\Gamma_C|^2}{1-|\Gamma_{B^+}|^2} P_A = 1{,}831\,mW$

RIVEDERE

Esercitazione 1

$\Omega = \frac{f}{f_c} = 1{,}5$; $\Omega - 1 = 0{,}5$; Butterworth $\longrightarrow N = 5$; $g_1 = g_5 = 0{,}6180$; $g_2 = g_4 = 1{,}6180$; $g_3 = 2$; $Z_0 = 50\,\Omega$.

$c_1 = \frac{g_1}{R_0}$; $l_2 = g_2 R_0$; $c_3 = \frac{g_3}{R_0}$; $l_4 = g_4 R_0$; $c_5 = \frac{g_5}{R_0}$;



$j\omega c_1 \longrightarrow j\frac{\omega}{\omega_0}c_1 \longrightarrow C_1 = \frac{c_1}{\omega_0}$; $C_n = \frac{g_n}{\omega_0 R_0}$;

$L_m = \frac{g_m R_0}{\omega_0}$;

$C_1 = 786{,}9\,fF$ $L_2 = 5{,}15\,nH$ $C_3 = 2{,}546\,pF$ $L_4 = 5{,}15\,nH$ $C_5 = 0{,}7869\,pF$ PROVARE

$h = \left(\frac{1}{8}\right)'' = \frac{2{,}54}{8 \times 100} = 3{,}175\,mm$; $\left.\frac{W}{h}\right|_{max(1)} = 4{,}724$; $\left.\frac{W}{h}\right|_{min(2)} = 0{,}1575$

$\varepsilon_{eff\,2} = 1{,}41$ $\beta_2 = \frac{2\pi f\sqrt{\varepsilon_{eff\,2}}}{c} = 73{,}83\,(min) \longrightarrow L \longrightarrow Z_{0om}\vartheta_m = Z_0 g_m$ $Z_{0om} = 130$

$\varepsilon_{eff\,1} = 1{,}44$ $\beta_1 = \frac{2\pi f\sqrt{\varepsilon_{eff\,1}}}{c} = 75{,}4 \Rightarrow C \longrightarrow Y_{0om}\vartheta_n = Y_0 g_n$ $Y_{0om} = \frac{1}{37}$

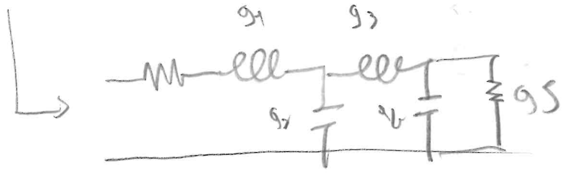
$l_L = \frac{g_m Z_0}{\beta_2 Z_{00}}$; $l_C = \frac{g_m Z_{00}}{\beta_1 Z_0}$ $l_1 = 6{,}065 = l_5$ $l_3 = 19{,}63\,mm$ ~~l_5~~ =

$l_2 = 6{,}44\,mm = l_4$;

RIVEDERE

2) $\frac{2,5}{1,65} = 0,515 + 1$; $\longrightarrow N = 4$; $g_1 = 3,4389$ $g_2 = 0,7483$ $g_3 = 4,3471$ $g_4 = 0,5920$ (3)

$g_5 = 5,8095$

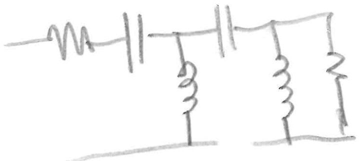


$j\omega\ell \longrightarrow -j\frac{\omega_0}{\omega}\ell = \frac{1}{j\omega C}$; $\frac{1}{C} = \omega_0 \ell$

$\Rightarrow C = \frac{1}{\omega_0 R_0 g_n}$; $j\omega c \longrightarrow -j\frac{\omega_0}{\omega} c =$

$= \frac{1}{j\omega L}$; $\frac{1}{L} = \omega_0 c = \omega_0 \frac{g_m}{R_0}$

$\hookrightarrow L = \frac{R_0}{\omega_0 g_m}$;



; $R_{out} = g_5 R_0$;

$\rightarrow C_1 = 0,37\,pF$ $L_2 = 4,254\,nH$ $C_3 = 0,293\,pF$ $L_4 = 5,377\,nH$ $R_5 = 290,5\,\Omega$

3) $f_0 = \sqrt{2,4 \cdot 2,5 \cdot 10^{18}} = 2,449 \cdot 10^9$; $\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$; $\Delta = 0,04082$; $\omega \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$;

$f_{R1} = 1,92\,GHz$; $f_{R2} = 3\,GHz$; @ f_{R2}, 10 ; @ f_{R1} $-12,5 = \Omega_1$

Ω_2 (= 10)

$\hookrightarrow$ @ $f_{R2} = 10$, $N = 2$ è ottimo!

Ora, ricavo le formule:

$\omega \longrightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$; $j\omega\ell \longrightarrow j\frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\ell =$

$= \underbrace{j\frac{1}{\Delta}\frac{\omega}{\omega_0}\ell}_{j\omega L} - \underbrace{j\frac{1}{\Delta}\frac{\omega_0}{\omega}\ell}_{\frac{1}{j\omega C}}$

$L = \frac{1}{\Delta}\frac{\ell}{\omega_0} = \frac{1}{\Delta}\frac{g_n R_0}{\omega_0} = \frac{g_n R_0}{\Delta\omega_0}$;

$\frac{1}{C} = \frac{\omega_0 \ell}{\Delta} \longrightarrow C = \frac{\Delta}{\omega_0 R_0 g_n}$

$j\omega c \longrightarrow \frac{1}{\Delta} j \frac{\omega}{\omega_0} c + \frac{\omega_0}{j\Delta\omega} c$;

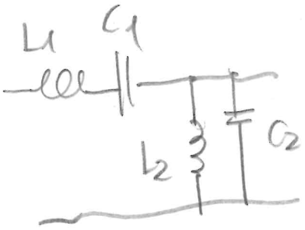
(jωC) (1/jωL)

$c = \frac{g_m}{R_0} \longrightarrow C = \frac{g_m}{R_0\omega_0\Delta}$ $\frac{1}{L} = c\frac{1}{\Delta\omega_0} \longrightarrow L = \frac{\Delta R_0}{g_m\omega_0}$

$\ell \longrightarrow \begin{cases} L = \frac{g_m R_0}{\Delta\omega_0} \\ C = \frac{\Delta}{\omega_0 R_0 g_m} \end{cases}$

$c \longrightarrow \begin{cases} L = \frac{\Delta R_0}{g_m\omega_0} \\ C = \frac{g_m}{R_0\omega_0\Delta} \end{cases}$

$\Longrightarrow$



$L_1 = 112,5\,nH$

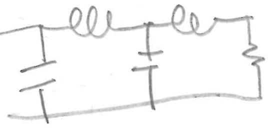
$C_1 = 37,51\,fF$

$L_2 = 93,78\,pH$

$C_2 = 45,02\,pF$

Esercitazione 3

1) $g_1 = g_4 = 0{,}7654$; $g_2 = g_3 = 1{,}8478$; →

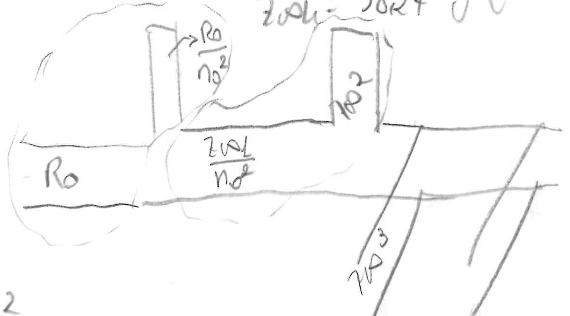


$j\omega L \rightarrow j\frac{\omega}{\omega_0} l \rightarrow L = \frac{l}{\omega_0} = \frac{g_m R_0}{\omega_0}$;

$j\omega C \rightarrow j\frac{\omega}{\omega_0} c \rightarrow j\omega C,\ C = \frac{c}{\omega_0} = \frac{g_m}{\omega_0 R_0}$

Richards: $Z_{0L} = g_m R_0$; $Z_{0C} = \frac{g_m}{R_0}$;

$Z_{01} = 65{,}33\,\Omega$
$Z_{02} = 32{,}39\,\Omega$
$Z_{03} = 27{,}06\,\Omega$
$Z_{04} = 38{,}27\,\Omega$



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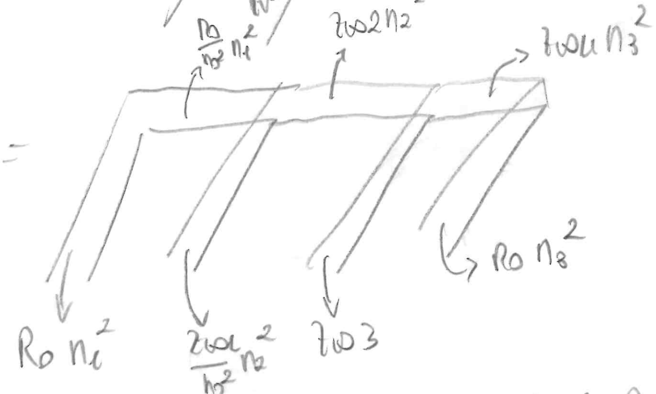


$n_3^2 = 1 + \frac{R_0}{Z_{04}} = 2{,}307$ $n_2^2 = \frac{\frac{Z_{01}}{n_0^2}}{Z_{02}} \cdot \frac{Z_{01}}{n_0^2 Z_{02}} + 1 = 1{,}307$

$n_0^2 = 1 + \frac{Z_{01}}{R_0} = 2{,}307$

$n_1^2 = \frac{R_0}{\frac{R_0}{n_0^2}} + 1 = n_0^2 + 1 = 3{,}307$

$R_0 n_1^2 = 165{,}3\,\Omega$ $Z_{02} n_2^2 = 120{,}7\,\Omega$ $R_0 n_3^2 = 115{,}3\,\Omega$

$\frac{R_0}{n_0^2} n_1^2 = 71{,}68\,\Omega$ $Z_{03} = 27{,}06\,\Omega$

$\frac{Z_{01}}{n_0^2} n_2^2 = 37\,\Omega$ $Z_{04} n_3^2 = 88{,}27\,\Omega$

165,3	0,2		$\sqrt{\varepsilon_{eff}}$
71,68	1,8		1,39
37	5		1,6
120,7	0,5		1,4
27,06	7		? 1,8
88,27	1		1,41
115,3	0,7		1,4
50	3		1,45

2) $\alpha = \alpha_c + \alpha_d$; $\alpha_c = \frac{R_s}{W Z_{00}}$, $R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}}$; $\rightarrow \alpha_c = 0,575$;

$$\begin{cases} R = Z_{00}\,\alpha\, l; \\ L = \frac{Z_{00}}{\omega_0}\frac{\pi}{4} \end{cases}$$

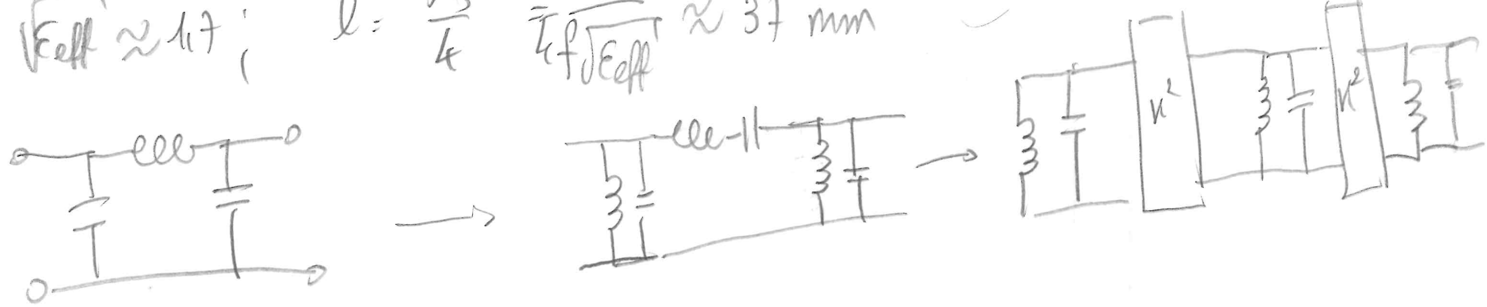
$\frac{W}{h} = 4,5$ [da grafico]

$\alpha_d = 29,33\,e{-3}$; $\alpha \approx 0,605$

$Q \approx \frac{\beta}{2\alpha}$

$\rightarrow \sqrt{\varepsilon_{eff}} \approx 1,7$; $l = \frac{\lambda_g}{4} = \frac{c}{4f\sqrt{\varepsilon_{eff}}} \approx 37\ mm$

3)



$$j\omega C \rightarrow j\frac{1}{\Delta}\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]C = \underbrace{\frac{1}{\Delta} j \frac{\omega}{\omega_0} C}_{j\omega C} - \underbrace{\frac{1}{\Delta} j \frac{\omega_0}{\omega} C}_{\frac{1}{j\omega L}} \rightarrow \frac{1}{L} = \frac{1}{\Delta}\omega_0 \frac{g_m}{R_0}$$

$$L = \frac{\Delta R_0}{\omega_0 g_m}$$

$$C = \frac{g_m}{R_0 \omega_0 \Delta}$$

$$C = \frac{\pi}{4 Z_{00} \omega_0} \rightarrow Z_{00} = \frac{\pi}{4 C \omega_0} = \frac{\pi R_0 \omega_0 \Delta}{4 g_m \omega_0} = \boxed{\frac{\pi R_0 \Delta}{4 g_m}}$$

Esercitazione 4 ①

$Z_{ri} = 50\ \Omega$;

$$\left.\frac{P_3}{P_2}\right|_{Tx} = \frac{|b_3|^2}{|a_2|^2}\,\frac{1-|\Gamma_3|^2}{1-|\Gamma_3|^2} \quad ; \quad \begin{cases} b_1 = S_{12}a_2 \\ b_2 = S_{23}a_3 \\ b_3 = S_{31}a_1 \end{cases} \quad ; \quad b_3 = S_{31}a_1 = S_{31}\Gamma_1 S_{12}a_2 ; \quad \left|\frac{b_3}{a_2}\right|^2 = |\Gamma_1 S_{12} S_{31}|^2 ;$$

Γ_3 è "noto";

$$\Gamma_2 = \left.\frac{b_2}{a_2}\right| ; \quad b_2 = S_{23}a_3 = S_{23}\Gamma_3 S_{31}a_1 = S_{23}\Gamma_3 S_{31}\Gamma_1 S_{12}a_2$$

$$\rightarrow \frac{P_3}{P_1} = |\Gamma_1 S_{12} S_{31}|^2 \frac{1-|\Gamma_3|^2}{1-|S_{23}\Gamma_3 S_{31}\Gamma_1 S_{12}|^2} \qquad \Gamma_3 = 0{,}238\ \exp(-j\ 0{,}966)$$

$\Gamma_1 = 0$ ⊕, 0 $0{,}2915\exp(j\ 0{,}1398)$

$$\frac{P_3}{P_1} = 0 \ ⊕ , \ 0 \quad 0{,}0852$$

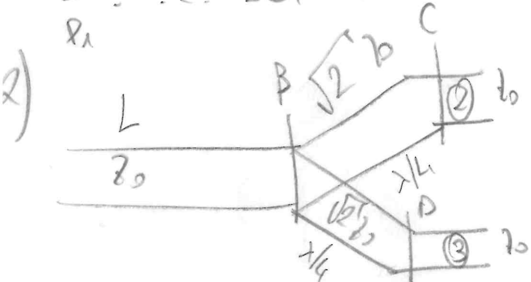
Per l'altro:

$$\left.\frac{P_3}{P_1}\right|_{Rx} = \frac{|b_3|^2}{|a_1|^2}\,\frac{[1-|\Gamma_3|^2]}{[1-|\Gamma_1|^2]} ; \quad \Gamma_1 = \frac{b_1}{a_1}; \ b_1 = S_{12}a_2 = S_{12}\Gamma_2 S_{23}a_3 = S_{12}\Gamma_2 S_{23}\Gamma_3 S_{31}a_1$$

$$\rightarrow \frac{b_3}{a_1} = S_{31} ; \longrightarrow \left|\frac{b_3}{a_1}\right|^2 = |S_{31}|^2 \frac{1-|\Gamma_3|^2}{1-|S_{12}\Gamma_2 S_{23}\Gamma_3 S_{31}|^2} ; \quad \Gamma_2 = 0{,}343\exp(+j\ 2{,}111);$$

$$\frac{P_3}{P_1} = \dots = 0{,}969.$$

2)



$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=a_3=0} = \frac{V_{C^+}^+}{V_{A^-}^+}$$

$$V_{C^+}^+ = V_{C^-}^+[1-\Gamma_{C^-}] = V_{B^+}^+\exp(-jkl)[1-\Gamma_{C^-}] =$$

$$= V_{B^-}^+[1-\Gamma_{C^-}]\frac{1-\Gamma_{B^-}}{1-\Gamma_{B^+}}\exp(-jkl) = V_{B^-}^+\zeta_{C^-}\cdot(-j) = -j\frac{1}{\sqrt{2}}\exp(\dots)$$

$$S_{11} = \Gamma_1|_{a_2=a_3=0} ; \ \Gamma_{C^-} = \frac{Z_0-\sqrt{2}Z_0}{Z_0+\sqrt{2}Z_0} = \frac{1-\sqrt{2}}{1+\sqrt{2}} = \frac{1-2\sqrt{2}+2}{1-2} = -(3-2\sqrt{2}) = 2\sqrt{2}-3 ; \quad \zeta_{C^-} = \frac{1}{\sqrt{2}} ;$$

$$\longrightarrow \zeta_{B^+C} = \sqrt{2} ; \quad Z_{B^+C} = \sqrt{2}\sqrt{2}Z_0 = 2Z_0 \longrightarrow Z_{B^-} = 2Z_0 \| 2Z_0 = 0. \quad S_{11} = 0 ;$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=a_2=0} ; \quad Z_{B^-} = 2Z_0 \| Z_0 = \frac{2Z_0^2}{3Z_0} = \frac{2}{3}Z_0 ; \ \zeta_{B^-} = \frac{\frac{2}{3}Z_0}{\sqrt{2}\,Z_0} = \frac{\sqrt{2}}{3} ; \quad \zeta_{C^+} = \frac{3}{\sqrt{2}} ;$$

$$Z_{D^+} = 3Z_0 ; \quad \Gamma_{C^-} = \frac{3Z_0 - Z_0}{3Z_0+Z_0} = \frac{1}{2} = S_{33} ;$$

$$S_{32} = \left.\frac{b_3}{a_2}\right|_{a_1=a_3=0} = \frac{V_{D^+}^+}{V_{C^+}^+} ; \quad V_{D^+}^+ = V_{D^-}^+[1+\Gamma_{D^-}] = V_{B^+D}^+\exp(-jkl)(1+\Gamma_{D^-}) = V_{B^-}^+\frac{1+\Gamma_{B^-}}{1+\Gamma_{B^+}}(1+\Gamma_{D^-})e^{-jkl} =$$

$$= [1+\Gamma_{B^-}]\,\zeta_{D^-}(-j)(-j)V_{C^+}^+ = -\frac{1+\Gamma_{B^-}}{1+\Gamma_{C^+}}(1+\Gamma_{C^-})\zeta_{D^-} = -\zeta_{D^-}\zeta_{B^-}(1+\Gamma_{C^-}) ;$$

$$\zeta_{D^-} = \frac{3}{\sqrt{2}} ; \ \zeta_{B^-} = \frac{\sqrt{2}}{3} ; \ \Gamma_{C^-} = \frac{1}{2} ; \longrightarrow S_{32} = S_{23} = -\frac{1}{2}.$$

RIVEDERE

b) Data $\underline{\underline{S}} = \begin{bmatrix} 0 & \frac{-j}{\sqrt{2}} & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ \frac{-j}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ $\frac{P_2}{P_1} = \left|\frac{b_2}{a_1}\right|^2 \frac{1-|\Gamma_2|^2}{1-|\Gamma_1|^2}$; $\Gamma_2 = 0$; $\Gamma_1 = S_{11}$;

$\frac{b_2}{a_1} = \; \rightarrow b_2 = S_{21} a_1 + S_{22} a_2 + S_{23} a_3$;

$\rightarrow \frac{b_2}{a_1} = S_{21}$; $\left|\frac{b_2}{a_1}\right|^2 = \frac{1}{2}$; $S_{11} = 0 \longrightarrow \frac{P_2}{P_1} = \frac{1}{2}$;

c) $Z_0 = 50\,\Omega$; $Z_{02} = Z_{03} = \sqrt{2}\,50\,\Omega = 70{,}71\,\Omega$;

dai grafici:

50	$\frac{W}{h} \approx 2{,}7 / 2{,}8$	$\sqrt{\varepsilon_{eff}} \approx 1{,}47$
70	$\frac{W}{h} \approx 1{,}7$	$\sqrt{\varepsilon_{eff}} \approx 1{,}42$

$\longrightarrow \frac{\lambda_g}{4}$, $\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}}$, $\lambda_0 = \frac{c}{f}$;

d) Si può rapidamente vedere che:

$P_2 = \frac{1}{2} Y_2 |V_2|^2$; $P_3 = \frac{1}{2} Y_3 |V_3|^2$; $\frac{P_2}{P_3} = 0{,}631 = \frac{Y_2}{Y_3}$; $\frac{1}{50} = Y_2 + Y_3$; $Y_2 = Y_3 \cdot 0{,}631$

$\rightarrow Y_3 \cdot 1{,}631 = \frac{1}{50} \rightarrow 81{,}55\,\Omega = Z_3$; $Z_2 = 129{,}2\,\Omega$;

3) $R = \frac{Z_0}{3} = 33{,}\bar{3}$; Ricordo che $\underline{\underline{S}} = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$\frac{P_3}{P_1} = \left|\frac{b_3}{a_1}\right|^2 \frac{1-|\Gamma_3|^2}{1-|\Gamma_1|^2};$$

ora: $b_3 = S_{31} a_1 + S_{32} a_2$; $a_2 = \Gamma b_2 = \Gamma [S_{21} a_1 + \cancel{S_{23} a_3}]$

$= S_{31} a_1 + S_{32} \Gamma_2 S_{21} a_1$; $\Gamma_3 = 0$;

$\Gamma_1 = \frac{b_1}{a_1}$; $b_1 = S_{12} a_2 + \cancel{S_{13} a_3}$ $a_2 = \Gamma_2 S_{21} a_1$

$\hookrightarrow \Gamma_1 = S_{12} S_{21} \Gamma_2$

$$\hookrightarrow \left.\frac{P_3}{P_1}\right|_1 = |S_{31} + S_{32} \Gamma_2 S_{21}|^2 \cdot \frac{1}{1-|S_{12} S_{21} \Gamma_2|^2}$$

Esercitazione 5 $h = \left(\frac{1}{8}\right)'' = \frac{2,54}{8 \times 100} = 3,175$ mm; (8)

1) 50 Ω ⟶ R = 100 Ω; Z = 70,7 Ω;

$l = \frac{\lambda_g}{4} = \frac{c}{4 f \sqrt{\varepsilon_{eff}}}$

R	w/h	$\sqrt{\varepsilon_{eff}}$	W	l (mm)
50 Ω	2,9	1,47	9,21 mm	42,53
~~100 Ω~~	~~0,8~~	~~1,4~~	~~2,54 mm~~	~~44,64~~
70,7 Ω	1,7	1,42	5,398 mm	44,01

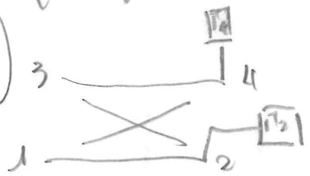
; $\underline{S} = -\frac{j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$; $\frac{P_2}{P_1} = \left|\frac{b_2}{a_1}\right|^2 \frac{1-|\Gamma_2|^2}{1-|\Gamma_1|^2}$;

$\frac{b_2}{a_1} = S_{12}$; $\Gamma_2 = 0$; $\Gamma_1 = \frac{b_1}{a_1} = 0$;

$\longrightarrow \frac{P_2}{P_1} = |S_{12}|^2$; $\frac{P_3}{P_1} = |S_{13}|^2$; $\frac{P_3}{P_2} = 0$; $\frac{P_3}{P_1} = |S_{31}|^2$;

$\eta_1 = 2\frac{P_2}{P_1} = 2|S_{21}|^2 = 1$;

$\eta_2 = \eta_3 = |S_{13}|^2 = \frac{1}{2}$;

2) 

$\underline{S} = \begin{bmatrix} 0 & 1 & 0 & j \\ 1 & 0 & j & 0 \\ 0 & j & 0 & 1 \\ j & 0 & 1 & 0 \end{bmatrix} \frac{1}{\sqrt{2}}$

$S_{11}^f = \frac{b_3}{a_1}$; $b_3 = S_{32} a_2 + S_{34} a_4$; $\begin{cases} a_2 = \Gamma_2 b_2 \\ a_4 = \Gamma_4 b_4 \end{cases} \to b_3 = S_{32}\Gamma_2 b_2 + S_{34}\Gamma_4 b_4$

$= S_{32}\Gamma_2[S_{21}a_1 + S_{23}a_3] + S_{34}\Gamma_4[S_{41}a_1 + S_{43}a_3] =$

$= a_1[S_{32}\Gamma_2 S_{21} + S_{34}\Gamma_4 S_{41}] + a_3[S_{32}\Gamma_2 S_{23} + S_{34}\Gamma_4 S_{43}]$

$b_1 = S_{12}a_2 + S_{14}a_4 = S_{12}\Gamma_2[S_{21}a_1 + S_{23}a_3] + S_{14}\Gamma_4[S_{41}a_1 + S_{43}a_3]$;

$\hookrightarrow a_1[S_{12}\Gamma_2 S_{21} + S_{14}\Gamma_4 S_{41}] + a_3[S_{12}\Gamma_2 S_{23} + S_{14}\Gamma_4 S_{43}]$

$\begin{matrix}1\\2\end{matrix}\begin{cases} b_1 = a_1[\Gamma_2 - \Gamma_4] + a_3[j\Gamma_2 + j\Gamma_4] \\ b_3 = a_1[j\Gamma_2 + j\Gamma_4] + a_3[jj\Gamma_2 + \Gamma_4] \end{cases} \to \underline{S} = \frac{1}{2}\begin{bmatrix} \Gamma_2 - \Gamma_4 & j[\Gamma_2+\Gamma_4] \\ j[\Gamma_2+\Gamma_4] & \Gamma_4 - \Gamma_2 \end{bmatrix}$

3) $\frac{V_4}{V_3} = \frac{V_4^+}{V_3^+} \frac{1+\cancel{\Gamma_4}}{1+\cancel{\Gamma_3}} = \frac{b_4}{b_3}$;

$b_4 = S_{42}\, a_2 + \cancel{S_{43}\, a_3}$; $a_2 = \Gamma b_2 \rightarrow b_4 = S_{42}\, \Gamma \left[S_{21}\, a_1 + \cancel{S_{24}\, a_4} \right]$

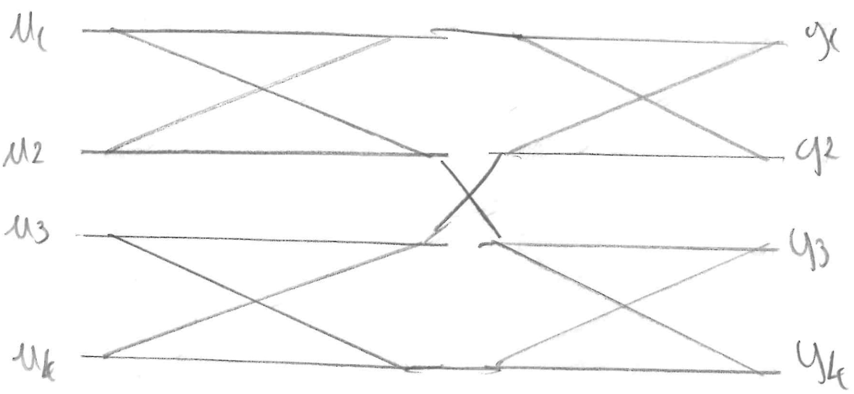
$\hookrightarrow b_4 = S_{42}\, \Gamma\, S_{21}\, a_1$; $b_3 = S_{31}\, a_1 + \cancel{S_{34}\, a_4}$

$\hookrightarrow \frac{b_4}{b_3} = \frac{S_{42}\, \Gamma\, S_{21}}{S_{31}} = \frac{V_4}{V_3}$; $\rightarrow \Gamma = \frac{V_4}{V_3} \frac{\cancel{S_{31}}}{\cancel{S_{42}}\, S_{21}} = \boxed{\frac{V_4}{V_3} \cdot \frac{1}{S_{21}}}$

$\Gamma = 0{,}8475 \angle \frac{\pi}{8}$

Esercitazione 6

Dato



$C_{dB} = -20 \log_{10}(\beta)$

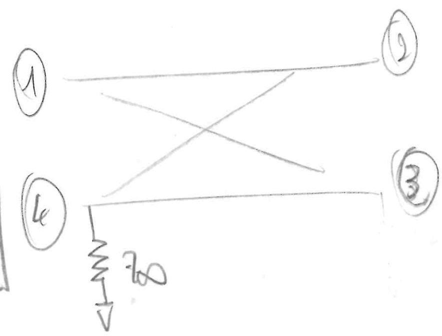
$\hookrightarrow \beta \simeq \frac{1}{\sqrt{2}}$; → 0,7161 ; → 0,6998 → $\alpha = 0{,}714$

$\alpha \simeq \frac{1}{\sqrt{2}}$;

$\alpha = 0{,}698$

Matrice "ideale" accoppiatore (considero accoppiatore BILANCIATO)

$$\underline{\underline{S}} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$



$b_2 = S_{21} a_1 + S_{24} a_4 =$

$= \alpha a_1 + j\beta a_4$

$b_3 = S_{31} a_1 + S_{34} a_4 =$

$= j\beta a_1 + \alpha a_4$

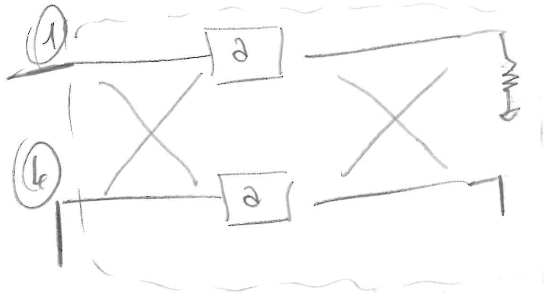
$y_1 = u_1 \alpha^2 + u_2 j\beta \alpha + u_3 \alpha j\beta + u_4 (j\beta)^2$ $\alpha = \sqrt{1-\beta^2}$;

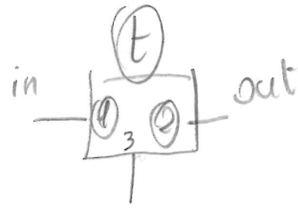
$= u_1 (\sqrt{1-c})^2 + j\sqrt{c}\sqrt{1-c}\, u_2 + u_3 \sqrt{1-c}\, j\sqrt{c} + (j\sqrt{c})^2 u_4$

Il peggio è $(\sqrt{1-c})^2 = (0{,}618)^2 = 0{,}4871$

3) Dato il blocco base:

$|S_{11}^a|^2 = 0{,}95$; $|S_{21}^a|^2 = 0{,}98$;





$$S^{acc} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

$$S_{11}^t = S_{12}^1 S_{11}^a S_{12}^1 + S_{13}^1 S_{11}^b S_{13}^1 = S_{11}^a \left[S_{12}^{1\,2} + S_{13}^{1\,2} \right] = S_{11}^a \left[\alpha^2 + (j\beta)^2 \right]$$

$$S_{21}^t = S_{12}^1 S_{11}^a S_{24}^1 + S_{13}^1 S_{11}^a S_{34}^1 = S_{11}^a \left[S_{12}^1 S_{24}^1 + S_{13}^1 S_{34}^1 \right]$$

$$S_{31}^t = S_{12}^1 S_{21}^a S_{13}^1 + S_{13}^1 S_{21}^a S_{34}^1 \quad = S_{11}^a [$$

$$= S_{21}^a \left[\alpha j \beta + \alpha j \beta \right] = 2j S_{21}^a \alpha \beta$$

Ora: se $\beta \cong 0{,}716$, $\alpha \cong 0{,}698$, $2\alpha\beta = 0{,}9997$; ≈ 1

se $\beta \approx 0{,}6998$, $\alpha \approx 0{,}7143$, $2\alpha\beta \approx 0{,}9998 \approx 1$

$\hookrightarrow \begin{cases} |S_{11}|^2 \cong |\Gamma|^2 \\ |S_{31}|^2 \approx |T|^2 \end{cases}$; $P_L = (|\Gamma|^2)^3 |T|^2 = 0{,}84$

$P_S = (|\Gamma|^2)^4 = 0{,}81$